Anonymity of Credit-Based Payment Systems*

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November 13, 2023

Abstract

Payment systems can be broadly categorized into two types: money-based payment systems (such as cash) that depend on the quality of the money exchanged, and credit-based payment systems (such as credit cards) that are based on the threat of punishment in case of default. It is commonly assumed that only money-based systems can provide anonymity because user anonymity prevents punishment. However, this is not necessarily the case. In a pseudonymous environment where agents use accounts (such as wallet addresses) to interact with each other, an account can provide a complete history of past actions without revealing the identity of its owner. Although individuals cannot be punished directly, accounts can face consequences such as loss of reputation. We demonstrate the existence of an anonymous credit-based payment system. However, we show that maintaining anonymity in such systems is costly.

Keywords: Payment Systems, Reputation, Credit, Anonymity

JEL Codes: E51, E42, L14, G19

*We thank Lukas Altermatt, Fernando Alvarez, Pierpaolo Benigno, Martin Brown, Harris Dellas, Lorenz Driussi, Janet Jiang, Charlie Kahn, Ricardo Lagos, Sebastian Merkel, Cyril Monnet, Dirk Niepelt, Remo Nyffenegger, Martina Pons, Randy Wright, and the participants of the Marcoeconomics Reading Group Bern, the Reading Group of the Economic Theory Group of the University of Basel, the Brown Bag Seminar Bern, the Young Economist Conference, the Rice-LEMMA Monetary Conference, the 3rd Annual CBER Conference, and the Summer Workshop on Money, Banking, Payments, and Finance for helpful comments and discussions.

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1 Introduction

Payment systems can be classified into two types: money-based or credit-based. Money-based payment systems involve the on-the-spot exchange of currency for goods and services, while credit-based payment systems entail the promise of future repayment. Throughout history, we have seen different forms of both types of payment systems. Money-based payment systems have evolved from the use of gold coins and paper money in the past, to the emergence of modern digital currencies like Bitcoin. These money-based payment systems can be costly because users have to hold low-return currency, but, in return, they can provide a high degree of anonymity.[1] This is because sellers of goods and services only care about the future exchange value of the money they receive, not whom they receive it from.[2] This is in stark contrast to credit-based systems which ranged from gift-giving practices to modern financial instruments such as cheques and credit cards. In credit-based payment systems promises, and not currencies, are offered in exchange for goods and thus its success depends on its ability "to ascertain the link between transactors and histories" (Kahn and Roberds (2009)).[3] This link is essential as it provides a mechanism to discourage deviant behavior, such as not paying one's credit card bill, by establishing clear punishments or consequences for non-compliance. With the recent surge in interest in cryptocurrencies and decentralized finance (DeFi[4]), which highlights a strong desire for anonymity in payment systems, a fundamental question arises: In the context of developing an anonymous payment system, can the payment system be credit-based or must it necessarily be money-based? This paper aims to improve understanding of anonymous payment systems and guide the development of future systems.

At the heart of the matter lies the question of what qualifies as anonymity. We distinguish two concepts of anonymity that we refer to as *strict anonymity* and *pseudonymity*. Under the former, a user is deemed anonymous if and only if the user's history of actions is private information. For instance, a payment system based on cash is considered strictly anonymous as there is generally no public record of cash transactions. In contrast, a user is pseudonymous if and only if the identity of agents responsible for certain actions is unknown even if the entire history of actions is publicly known. The scenario we have in mind is one in which agents use *accounts* (or pseudonyms) when transacting with each other.[5] Moreover, all transactions

[1] The classical example is cash: it offers high anonymity but it is costly to hold because it does not pay any interest compared to other (nominal) assets (given we do not implement the Friedman Rule). This goes back to at least Friedman (1969) and has been formalized in various frameworks such as the money-in-the-utility-function model (Sidrauski, 1967), the cash-in-advance model (Lucas and Stokey, 1987), or more recent search models (Lagos and Wright, 2005).

[2] It is important to stress that money-based payment systems *can*, but *must not* be, anonymous. For example, compared to cash, which offers a high degree of anonymity, Bitcoin has a relatively low degree of anonymity (Reid and Harrigan, 2013).

[3] In the presence of intermediaries the link is somewhat different. Instead of paying the seller with a promise, the intermediary, such as a credit card company, pays the merchant with tokens and receives a promise from the buyer. Thus, in this case, the promise issued is to the intermediary, and not the seller directly, but the idea remains the same.

[4] A comprehensive overview of DeFi and how it compares to traditional (centralized) finance is provided in Qin et al. (2021).

[5] Examples of pseudonyms are aplenty: email accounts, user names, gamer tags or wallet adresses just to name a few.

are recorded and made public but the ownership of accounts is not known such that everything that account x has done is known except the identity of the owner of account x. For example, a payment system such as Bitcoin is pseudonymous since each transaction is recorded on the blockchain but transactions are not signed with someone's real name but with a pseudonym (the public key).[6]

It is clear that credit, almost by definition, is not feasible under strict anonymity as tracking *who owes whom* requires some knowledge of past actions.[7] However, whether or not credit, and in particular a credit-based payment system, is possible under pseudonymity is still an open question both in terms of its *economic* and *technical* feasibility. For a credit-based payment system to attain technical feasibility under pseudonymity, it necessitates the payment technology to a) record all debt obligations between accounts, b) validate the fulfillment of these obligations, and c) establish a transparent and easily accessible public record of these transactions. With the rapid advancements in blockchain technology in recent times, the prospect of achieving this feasibility within the upcoming years appears promising, despite some challenges. While its technical feasibility is an interesting question, this paper takes the technical feasibility as given and investigates its economic feasibility. The lack of research on this question is surprising since many areas of the internet operate under a pseudonymous regime where users interact with each other through website accounts, blockchain wallets, or virtual avatars, while a large part of the user's activity is recorded and observable. The goal of this paper is therefore to study the economic feasibility of credit in the context of pseudonymous, and more generally anonymous, payment systems taking the technical feasibility as given. This, to our knowledge, novel approach of pseudonymity represents a middle ground between the two extremes often considered in the literature: strict anonymity (which excludes credit-based systems) and full information (where anonymity concerns are absent).

To pursue our goal, we want to employ a model that a) incorporates some of the relevant frictions that plague actual payment systems such as lack of double coincidence of wants, limited commitment, and costly interactions between people, b) is simple enough to yield reduced form results and c) is familiar to those in the monetary theory and payment literature. A suitable candidate is therefore the framework based on Lagos and Wright (2005), and more specifically Rocheteau and Wright (2005), which have been widely used to address a broad set of questions.

Our model, which is populated by two types of agents called buyers and sellers, has a simple structure which, even though omitting many features of actual payment systems, captures the main economic challenges.[8] Each period is divided into two subperiods: In the first subperiod (DM), buyers probabilistically

[6]See Schär and Berentsen (2020) for a detailed description of cryptocurrencies and Bitcoin in particular.

[7]Kocherlakota and Wallace (1998) show that under strict anonymity, an optimal payment system must be money-based.

[8]For those not familiar with the work of Lagos and Wright (2005) this terminology might be confusing. Alternatively, one can think of buyers as borrowers and sellers as lenders.

match with sellers in bilateral meetings where only sellers can produce goods which buyers desire to consume. In the second subperiod (CM), buyers will be producers for sellers. Even though both parties benefit from this intertemporal trade, i.e. sellers produce for buyers in the DM and buyers produce for sellers in the CM, the lack of commitment introduces significant trade frictions. A successful payment system needs to overcome these frictions, i.e. provide the incentives that buyers produce for sellers in the CM. Different from other models in the literature, our model employs a novel record-keeping technology that perfectly tracks debt relations and repayment histories across different accounts, which are owned by buyers. Buyers can always create new accounts at zero cost and choose each period on which account to record their actions. This record-keeping technology aligns with the previously mentioned notion of pseudonymity. Finally, to make the problem interesting, we make the assumption that there is a constant flow of new buyers entering the market, who have no prior history of transactions. Thus, at any given time, an account lacking any history could indicate either a "young" buyer, opening an account for the first time, or an "older" buyer who has opened a new account, possibly due to defaulting on a previous account in an attempt to conceal her negative history. The challenge of establishing a credit-based payment system in such an environment is thus the following: How to establish punishment for buyers who have defaulted on their debt if they can always "clear their history" by creating and using new accounts?

Our main finding is that there always exists a credit-based equilibrium where agents are pseudonymous. Intuitively, those equilibria work in the following way: buyers using accounts earn *reputation*, which is a mapping from the account's history, by consistently repaying their debt, which increases the size of debt the account is allowed to issue, and subsequently the amount the buyer can consume, in the future. Accounts that have a history of defaulting on their debts are barred from using borrowing in the future. Even though buyers can always create new accounts and borrow again after defaulting, this is costly to do because newly created accounts have no reputation and can therefore borrow only little. In these equilibria, buyers never endogenously default on their debt because the *value of reputation*, i.e. the difference in continuation values between an account with a given level of reputation compared to an account with no reputation, is sufficiently high. An important, and somewhat surprising, implication of such equilibria is that buyers optimally use only one account despite having the option to use multiple accounts.[9] The rationale behind this is that using a second account incurs an opportunity cost of forfeiting the chance to accumulate even more reputation on the primary account given that buyers only have one match with one seller per period.

Moreover, we can ask what is required for a credit-based payment system to work. First, accounts that have defaulted have to be punished by reducing the amount they can borrow in the future (in our case,

[9] The fact that buyers can always create new accounts has still an important effect on the equilibrium as it lowers the cost of defaulting.

we assume that no amount can ever be borrowed again). Second, accounts that issue more debt than the equilibrium amount (as a function of the account's reputation) must be similarly reprimanded. The reason is that absent such a punishment, buyers and sellers bilaterally agree to exchange the highest amount of debt such that the buyer is indifferent between repaying and defaulting. However, those "not-too-tight" debt limits, using the language of Alvarez and Jermann (2000), cannot be part of a credit equilibrium. The reason is an externality: when bargaining buyers and sellers do not internalize how their choices affect the value of reputation in equilibrium. We then show that if debt limits are not-too-tight, then the value of reputation, and therefore debt limits, collapse to zero. Therefore, by punishing deviation from the equilibrium amount, we can enlarge the set of incentive-feasible credit equilibria and construct equilibria where the value of reputation and debt limits are positive.[10]

Finally, while our model shows that a credit-based payment system is always economically feasible, we show that maintaining credit in a pseudonymous environment is costly. This is due to a trade-off between the consumption of "older" buyers (those with a lot of reputation) and the consumption of "younger" buyers (those with little or no reputation). Intuitively, supporting a large volume of trade requires a high value of reputation to keep buyers from defaulting on their debts. But for reputation to be highly valued, it must be that consumption is restricted for those agents with little reputation, such as "young" buyers. Generally, pseudonymity is costly because it is impossible to differentiate between "young" buyers and those who have defaulted, and, as a result, any punishment scheme affects buyers not only off-equilibrium (if they default) but also on-equilibrium (when they are "young").

Literature Review

Our study is situated within the literature that microfounds the usage of money in search-theoretic environments. Seminal work in this area includes Kiyotaki and Wright (1989), Trejos and Wright (1995), and Lagos and Wright (2005), who discuss the different conditions under which money can exist and is essential.[11] Other researchers who build on their work and introduce credit include Telyukova and Wright (2008), Lotz and Zhang (2015) and Gu et al. (2016). A comprehensive review of this literature can be found in Lagos et al. (2017). The main conclusion from this body of work is that for money to be essential, the environment must have certain frictions, such as limited commitment, lack of double coincidence of wants, and anonymity, which makes it impossible to support credit. However, as we have already mentioned above, this literature applies a stricter definition of anonymity than we do in this paper.

[10] Bethune et al. (2018) show that punishing out-of-equilibrium behavior can generate a much larger set of equilibria than those that are not-too-tight.

[11] Essentiality is defined in Nosal and Rocheteau (2017): "Money is essential if the set of allocations that is feasible with money is larger than the set of allocations feasible without money and if it contains at least one allocation that is socially preferred".

A particularly relevant paper to our question is Araujo (2004) in which they show that money is not necessarily essential even if agents are anonymous in the stricter sense. They demonstrate that “word-of-mouth contagion” can maintain a gift-giving equilibrium. However, as they show, this only works if agents are sufficiently patient and the population is sufficiently small. Our equilibrium exists for infinitely many agents with arbitrary discount factors.

Another line of work related to ours is Kocherlakota (1998) and Kocherlakota and Wallace (1998), in which they argue that the primary role of money is to serve as a crude “store of memory”. Our work can be seen as a modification of their work where agents have a strong preference for (pseudo-) anonymity, which is completely absent from theirs. In this sense, our work agrees with their assessment that “money is memory”, but we also emphasize that “money is anonymity”, since there are also costs in maintaining anonymous credit-based systems.

We are also not the first to think about accounts or pseudonyms. Friedman and Resnick (2004) discuss this in the context of cooperation. In particular, they study an infinitely repeated prisoner’s dilemma where, similar to our model, agents use accounts. They conclude, as we do, that maintaining cooperation is costly as building up reputation is costly. Nevertheless, they miss many of the specifics of payment systems in their treatment, which we think are important to highlight.

Wang and Li (2023) also study credit equilibria in an pseudonymous context in a similar environment. They share some similar results but there are also some significant differences. First of all, they either restrict agents to only have one active account or impose some cost for holding several accounts. Our study gives the agents more freedom to hold several accounts at the same time without imposing such restrictions. We show that such restrictions are not necessary to generate pseudonymous credit equilibria. Second, we show that credit equilibria can only exist if debt limits are not-too-tight, a result absent from their analysis. Third, we proof existence of the credit-based payment system.

Finally, the work of Kehoe and Levine (1993) and Alvarez and Jermann (2000) shares with our work the feature that endogenous debt limits arise in equilibrium. Agents lack commitment, and they can thus only credibly commit to a certain amount of debt. However, it is hard to incorporate money into their model due to the centralized structure of trade.

Structure

The paper is structured as follows: Section 2 sets out the general environment, preferences and technology. Section 3 describes the equilibrium recursively and proofs the existence of an credit-based anonymous payment system.

2 The Environment

Time is discrete and each period is given by $t = 0, 1, \ldots \infty$. There are two types of agents called *buyers* and *sellers* of which there is a $[0, 1]$ continuum of each type.

Each period consists of two sub-periods. During the first sub-period, agents enter the *decentralized market* (DM) where buyers match with a random seller with probability σ. Sellers can produce and sell the *DM-good* (or trade), $q_t \in \mathbb{R}_0^+$, which the buyers can buy and consume. The terms of trade are determined by bargaining. At the end of the DM buyers exit the economy randomly with probability p and are immediately replaced by new buyers entering the economy so that the mass of agents remains unity.[12] In the second sub-period, agents trade in the *centralized market* (CM), in which agents trade "along a budget constraint" and market clearing determines prices. In this market, agents may either consume the *CM-good*, $x_t \in \mathbb{R}_0^+$, or supply labour, $y_t \in \mathbb{R}_0^+$. There exists a linear technology that maps labour into the CM-good, $x_t = f(y_t) = y_t$. Both the CM- and DM-good are non-storable across sub-periods.

The buyer and seller's expected lifetime utility, respectively, is given by $U_t^b = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[u(q_{t+j}) + x_{t+j} - y_{t+j}]$ and $U_t^s = \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[-c(q_{t+j}) + x_{t+j} - y_{t+j}]$ where $u(q)$ is $\mathcal{C}^2$ and satisfies $u'(q) > 0$, $u''(q) < 0$, $u(0) = 0$, $\lim_{q\to\infty} u'(q) = 0$ and $\lim_{q\to 0} u'(q) = \infty$. Similarly, $c(q)$ is $\mathcal{C}^2$ and satisfies $c'(q) > 0$, $c''(q) > 0$, $c(0) = 0$, $\lim_{q\to\infty} c'(q) = \infty$ and $\lim_{q\to 0} c'(q) = 0$. Finally, we can denote the *real interest rate* as $r \equiv \frac{1}{\beta(1-p)} - 1 \in (0, 1)$.[13]

When buyers and sellers are in a match during the DM, they have the opportunity to make the following trade: the seller produces DM-goods for the buyer on the spot and, as a compensation, the buyer promises to produce CM-goods for the seller in the subsequent sub-period. Buyers have an incentive to make this trade because they can consume the DM-good on the spot. The sellers, on the other hand, are promised $b_t \in \mathbb{R}_0^+$ units of the CM-good by the buyer. Furthermore, the terms of trade, i.e. how much goods are produced against how many promises, is determined by bargaining. We assume the proportional bargaining solution where $\theta \in (0, 1)$ denotes the buyer's bargaining power.[14] Finally, let us define the *first-best trade* as $q^* = \arg\max_q [u(q) - c(q)]$.[15]

We make the following assumption on the informational structure of the economy: all current and past actions are perfectly observable by anyone but there is limited knowledge about who committed which actions.

[12] The "perpetual youth" structure has its origins in Blanchard (1985).

[13] The real interest is the rate at which agents are willing to shift one unit of consumption one period in the future. Due to the quasi-linearity of preferences, agents are only willing to transfer goods if their compensated for their effective discount rate $\beta(1 - p)$.

[14] The proportional bargaining solution is due to Kalai (1977) which provides axiomatic foundations. Strategic foundations in the context of search models are given by Hu and Rocheteau (2020).

[15] Due to the linearity of the CM-good, there are no gains from trade in producing the CM-good. Nevertheless, the CM-good is useful because it serves to settle the debt the buyer incurs during the DM.

To be precise, buyers use an *account*[16], $a \in \mathcal{N}$, when interacting with a seller and all actions undertaken with a given account are perfectly observable.[17] Let us denote $\xi_t^a = \{q_j^a, b_j^a, x_j^a, y_j^a\}_{j=0}^t$ as the *history of past actions* (or memory) which records all actions undertaken by account a up to period t. Importantly however, the ownership of these accounts is private information and buyers may create and use as many accounts as they wish. Buyers can create new accounts during each CM at zero cost. As a result, buyers can always hide part of their history by creating and using another account.

3 Recursive Equilibrium

We will now describe the equilibrium in recursive form. Observe that the state variables in each period include the history of past actions of every account. Combined with the fact that this is an incomplete information environment implies that there are potentially many equilibria to consider. We will therefore restrict our attention to a particular subset of equilibria. First of all, only pure-strategy equilibria are considered. Second, we introduce *reputation* which is a mapping from the history of past actions to a natural number. We will study equilibria where reputation is a sufficient statistic for the history of past actions in the sense that the equilibrium can be described completely by knowing merely the reputation of each account.[18] While one can conceive of many different mappings, we construct our equilibrium with the following properties: account a's reputation in period t, $n_t^a \in [-1, 0, 1, \ldots, N]$, increases by one whenever an account makes a promise and fulfils it (i.e. the account repays all his debts issued during the DM of the same period). Additionally, we assume that there is some maximum level of reputation $N \in \mathbb{N}$ that can be achieved. The account is marked as a *deviator*, or $n_t^a = -1$, whenever it does not repay his debts (i.e. not produce the promised amount of CM-goods) or if the amount of debt issued exceeds the equilibrium amount of other accounts with the same history. Therefore, let us denote $\hat{b}(\xi)$ as the equilibrium amount of debt given history ξ. Accounts which are marked as deviators will be punished (see below for details). This will serve two purposes: first it makes defaulting costly and second it introduces a punishment for "excessive borrowing" and will expand the set of possible credit equilibria.[19] In particular, we will see that absent this second punishment mechanism, debt limits cannot be positive in the equilibria we are considering. Third, sellers do not trade with accounts which are marked as deviators. Given that sellers believe that other sellers refuse to trade with deviators, it is in fact optimal for them to do so as well. The reason is straightforward: the buyer's incentive to honour their debt is to avoid being marked as deviator. As a result, accounts already marked as deviators have "nothing to lose" and will therefore default on any promise. Sellers would

[16] Equivalently, one could use the term *pseudonym*. Given the topic at hand, account might be more topical though.

[17] We assume that accounts are unique. Hence, it is not possible to imitate another agent by taking on his history.

[18] Of course, there might be other equilibria in which reputation is not a sufficient statistic.

[19] This point was forcefully made in Bethune et al. (2018).

anticipate this and therefore refuse to trade with a deviator. Fourth, and following up on the previous point, it follows that accounts marked as deviators are never used again. Since accounts are costless to create, it is consequently (weakly) optimal to create a new account whenever an account is marked as deviator. We thus use the following simplifying notation when talking about the buyer's state variables (the reputation of the accounts): whenever an account is marked as deviator, we will write $n_t^a = 0$ instead of $n_t^a = -1$. This is simply shorthand for saying that the agent "replaces" the now-useless account with a new one with an empty history of past actions (and therefore zero reputation). Formally, we can define a *reputation system* as some N and where reputation is defined as:

Definition 1 (Reputation) *For a given N, account a has reputation $n_t^a =$ $\min\{\sum_{j=0}^{t} \mathbb{I}[b_j^a > 0], N\}$ if and only if $b_j^a \leq (y_j^a - x_j^a)$ and $b_j^a \leq \hat{b}(\xi^a)$ for all $j \leq t$ and $n_t^a = 0$ else.*

Fifth, we restrict ourselves to a *monotonic* reputation system which means that the implied amount of trade is weakly increasing in reputation, $q_{t,n} \geq q_{t,n-1}$ $\forall n > 0$. Sixth, we will simplify the problem minimally by making the following assumption: buyer's can only hold maximally two accounts with positive reputation simultaneously. This reduces notation and simplifies one of the proofs. However, we do not consider this a serious restriction. Because, as we will show below, buyers will not have an incentive to hold a second account, we don't see any reason why this would change given they could hold more than two accounts simultaneously.[20] Seventh, bargaining solutions only depend on buyer's current account history, which is observable to the seller, and not on any private information including the histories of buyer's other accounts. Finally, time indexes will be suppressed unless unclear and let us denote any generic variable x by x_n to denote it being conditional on reputation n.

3.1 The Centralized Market

Let us consider a buyer with accounts (n_1, n_2) who used, without loss of generality, account n_1 to make promises $b \geq 0$ in the previous DM. Three subcases must be considered: $b = 0$, $\hat{b}_{n_1} \geq b > 0$ and $b > \hat{b}_{n_1}$. The CM-value function for the first case, where no debts where issued, is given by:

$$W^b(0, n_1, n_2) = \beta V^b(n_1, n_2) \tag{1}$$

[20] To be redundant but clear: we are not restricting how many accounts buyers use over their lifetime. Rather the restriction is on the simultaneous holding of more than two accounts with positive reputation. This is similar to Wang and Li (2023) but while they only allow agents to hold one account for positive reputation, we allow for two making the problem much more delicate and tricky to handle.

where $V^b(n_1, n_2)$ is the buyer's DM-value function.[21] This corresponds to the case were a buyer did not meet any sellers. As as result, there are no debts to repay and no reputation to be gained (or lost). The more interesting case is where agents enter the CM with debts that do not exceed the equilibrium amount given reputation n_1, i.e. $\hat{b}_{n_1} \geq b > 0$:

$$
\begin{aligned}
W^b(b, n_1, n_2) = &\max_{x,y,\eta} x - y \\
&+ \beta[\eta V^b(\min(N, n_1+1), n_2) + (1-\eta)V^b(0, n_2))] \\
&\text{s.t. } \; x + \eta b = y
\end{aligned}
$$

where $\eta = \eta(b, n_1, n_2) \in \{0, 1\}$ is the optimal choice that a buyer with accounts (n_1, n_2) and debt b will repay its debt. During the CM, buyers decide how much to consume or produce of the CM-good and decide whether they repay their debt or default. If they repay, they incur linear costs $(y - x = b > 0)$ to produce for sellers and their first account will gain reputation (unless reputation is already at maximum reputation). If they decide to default, then no costs are incurred $(y - x = 0)$ but the agent's first account will loose it's reputation such that $n_1 = 0$ in the next period. By inserting the budget constraint into the objective function we can simplify the CM-value function given $\hat{b}_{n_1} \geq b > 0$:

$$
W^b(b, n_1, n_2) = \max_{\eta} -b\eta + \beta[\eta V^b(\min(N, n_1+1), n_2) + (1-\eta)V^b(0, n_2)]. \tag{2}
$$

For the third case, $b > \hat{b}_{n_1}$, the Bellman equation can similarly be written as:

$$
W^b(b, n_1, n_2) = \max_{\eta} -b\eta + \beta V^b(0, n_2). \tag{3}
$$

According to definition (1), a buyer is marked as deviator if $b > \hat{b}_{n_1}$. Hence, the buyer loses his reputation irrespective if he repays his debt or not. Trivially then, buyers would always default in that case: $\eta(b, n_1, n_2) = 0$ for all $b > \hat{b}_{n_1}$. If however $\hat{b}_{n_1} \geq b > 0$ then, from equation (2), we can conclude that $\eta(b, n_1, n_2) = 1$ if and only if the following *no-default*(ND)-constraint is satisfied:

$$
\beta(V^b(\min(N, n_1+1), n_2) - V^b(0, n_2))] \geq b. \tag{4}
$$

We define the *debt-limit* as $B(n_1, n_2) \equiv \beta(V(\min(N, n_1+1), n_2) - V(0, n_2))]$ which is the value above which a buyer with accounts (n_1, n_2) will default with probability one. Hence, buyers only repay their debt if the

[21] Here we implicitly assume that $y = x = 0$ which is optimal due to the linearity of the CM-good utility and production cost.

amount of debt is less or equal than the cost of losing the account's reputation.

Similarly, we denote the CM-value function of a seller who enters the CM with debts b issued by an account with reputation n_1 by:

$$W^s(b, n_1) = \mathbb{E}[b \mid n_1] + \beta V^s. \tag{5}$$

where $\mathbb{E}[b \mid n_1] = b(1-p)\sum_{n_2=0}^{N} \eta_t(b, n_1, n_2)\alpha_t(n_2 \mid n_1)$ is the conditional value of debts b issued by an account with reputation n_1 and $\alpha_t(n_2 \mid n_1)$ is the conditional probability that an account with reputation n_1 holds a second account with reputation n_2.[22] Intuitively, sellers consume the CM-good by redeeming their debts with buyers. However, as buyers may default, sellers can only redeem the promises with probability $(1-p)\sum_{n_2=0}^{N} \eta_t(b, n_1, n_2)\alpha_t(n_2 \mid n_1)$ where $(1-p)$ buyers do not default exogenously and $\sum_{n_2=0}^{N} \eta_t(b, n_1, n_2)\alpha_t(n_2 \mid n_1)$ of buyers who have promises b with reputation n_1 do not default endogenously. Importantly, because a buyer's reputation on their second account is unobservable to the seller, sellers cannot perfectly anticipate a buyers default. They must therefore form expectations about the probability of an endogenous default.

3.2 Terms of Trade

The bargaining solution is based on Kalai (1977). Buyers maximize their trade surplus while ensuring that the seller's expected trade surplus is equal to a proportional share $1-\theta$ of the total surplus. The bargaining problem between a seller and a buyer with accounts n_1 and n_2 can then be written as:

$$\begin{aligned}
&\max_{q,b} u(q) + (1-p)(W^b(b, n_1, n_2) - W^b(0, n_1, n_2)) \\
&\text{s.t. } W^s(b, n_1) - W^s(0, n_1) = \theta u(q) + (1-\theta)c(q) \ \forall n_1.
\end{aligned}$$

Using equations (2) and (5), the problem can be written as:

$$\begin{aligned}
&\max_{q,b} u(q) - \eta(b, n_1, n_2)(1-p)b \\
&\text{s.t. } \mathbb{E}[b \mid n_1] = \theta u(q) + (1-\theta)c(q) \ \forall n_1.
\end{aligned} \tag{6}$$

As we will study an equilibrium where the bargaining outcome will on depend on the reputation of the account that is being used, we will denote its solution as q_n and b_n (more on that below).

[22] We do not further describe the distribution of accounts as it will not be of relevance in describing the equilibrium. On an intuitive level, the distribution matters in so far as the "default premium" charged by sellers depends on this distribution. As we study an equilibrium without endogenous default, we can ignore it. However, it can be characterized fairly easy.

3.3 Decentralized Market Value Function

The value function for a buyer entering the DM with accounts n_1 and n_2 can be written as:

$$\begin{aligned} V^b(n_1, n_2) =& \sigma \max\{u(q_{n_1}) + (1-p)W^b(b_{n_1}, n_1, n_2), u(q_{n_2}) + (1-p)W^b(b_{n_2}, n_1, n_2)\} \\ &+ (1-\sigma)(1-p)W^b(0, n_1, n_2) \end{aligned} \tag{7}$$

where q_n and b_n solve the bargaining problem, (6). When agents enter the DM they either meet a seller (first line) or they meet no one (second line). If they meet a seller, they may choose which account they will use. As seen in the bargaining problem, the choice of account influences the terms of trade as different accounts may have different levels of reputation. Finally, observe that the buyer only enters the CM if he does not exit the economy, which occurs with probability $(1-p)$. From equation (7) one can also infer that a buyer never uses a second account while the first one has positive reputation if

$$u(q_n) + (1-p)W^b(b_n, n, 0) > u(q_0) + (1-p)W^b(b_0, n, 0) \ \forall n > 0. \tag{8}$$

In simple terms, if using an account with positive reputation yields more instantaneous utility from consuming and a higher continuation value compared to using an account with zero reputation, a second account is never used.[23] The DM function for a seller can be written similarly:

$$V^s = \sigma \mathbb{E}_n[-c(q_n) + W^s(b_n, n)] + (1-\sigma)W^s(0, 0)$$

where q_n and b_n solve the bargaining problem, (6). Sellers may meet a buyer or not. If they do, then they meet a random buyer with random reputation and produce goods for the buyer depending on their reputation.

3.4 Equilibrium

Now we are able to define the equilibrium. In particular, we are interested in studying a *stationary*, *symmetric* and *single-account* equilibrium. To be precise, we want to study an equilibrium which has a stationary distribution (with respect to accounts), strategies that are symmetrical across all agents and buyers never want to use a second account, i.e. $\alpha(n_2 = 0 \mid n_1) = 1$ for all n_1.[24]

[23] At first, this may seem to be obvious since we assumed that more reputation allows for more trade. However, the issue is more subtle as buyers could use the second account to eventually default on his promise while using the primary account as a "back-up". As we will see below however, in our equilibria this cannot occur.

[24] Our equilibrium definition is based on the concept of a perfect Bayesian equilibrium. That is, our equilibrium will be defined by strategies and beliefs such that strategies are sequentially rational and beliefs are derived, if possible, by Bayes rule. See Mas-Colell et al. (1995) for a formal definition

Definition 2 (Pure-Reputation Equilibrium) *A stationary*[25] *and symmetric equilibrium where* $\alpha(n_2 = 0 \mid n_1) = 1$ *for all* n_1 *is given by DM-consumption* $\{q_n\}_{n=0}^{N}$ *and debts* $\{b_n\}_{n=0}^{N}$ *such that*

1. *the buyer's CM-value functions are respectively given by (1), (2) and (3),*
2. *the terms of trade solve the bargaining problem (6),*
3. *the buyer's DM-value function is given by (7),*
4. *buyers never use the second while the first account has reputation, (8),*

for a given monotonic reputation system with maximum reputation N.

We proceed as follows: First, we can solve the bargaining problem. Second, we then proceed to show that this implies that agents will never use the second account while the first one has reputation. Finally, we show that in equilibrium no one defaults and buyers only ever use one account.

Given that $\alpha(n_1 \mid n_2) = 0$, the bargaining problem between a seller and a buyer with no second account can be simplified to:

$$\max_{q,b} u(q_n) - (1-p)\eta(b,n,0)b_n \tag{9}$$

$$\text{s.t. } (1-p)\eta(b,n,0)b_n = \theta u(q_n) + (1-\theta)c(q_n) \equiv \omega(q_n)\ \forall n$$

As shown in the Appendix, the solution to this problem is given by:

$$q_n = \left\{ \begin{array}{ll} q^* & \text{if } \min\{B(n,0), \hat{b}_n\} \geq c(q^*) \\ \omega^{-1}(\min\{B(n,0), \hat{b}_n\}) & \text{if } \min\{B(n,0), \hat{b}_n\} < c(q^*) \end{array} \right\} \tag{10}$$

and

$$b_n = \frac{\omega(q_n)}{(1-p)}.$$

The solution has an intuitive interpretation: Optimally, buyers and sellers want to trade q^* in order to maximize the total trade surplus (which they then distribute according to θ). However, because of limited commitment the buyer may not be able to compensate the seller sufficiently, as the buyer can maximally promise $\min\{B(n,0), \hat{b}_n\}$ without defaulting. Remember, buyers with an account with reputation n (and no second account) would default if their promises exceeded this threshold and, as a result, sellers would never accept this. If buyers can credibly promise more than $\omega(q^*)/(1-p)$ then q^* is traded. If not, then buyers

[25] Formally, it means that $\alpha_t(n_1 \mid n_2) = \alpha(n_1 \mid n_2)$ for all n_1, n_2.

promise as much as possible and receive $\omega^{-1}(\min\{B(n,0),\hat{b}_n\})$ in DM goods. Therefore, for all levels of reputation on the buyer's first account, all allocations must satisfiy the following no-default constraints:

$$\beta(1-p)(V^b(\min(N,n+1),0)-V^b(0,0))] \geq \omega(q_n)\ \forall n. \tag{11}$$

Next, we can derive the following result:

Proposition 1 *If $\alpha(n_1 \mid n_2)=0$ then (8) is always satisfied.*

The proof for this result can be found in the Appendix. The proposition says that if sellers belief that there are no buyers holding second accounts then buyers have, in fact, no incentive to ever use a second account. To understand the intuition behind it, observe that in equilibrium sellers expect buyers to use only one account. As a result, in equilibrium, sellers can accurately predict whether a buyer with a single account will default or not. Defaulting with a single account can therefore never be profitable. But the question is: why wouldn't a buyer create a second account to deceive the seller and default at some later stage? But, as we show in the proof, nothing can be gained from such a strategy because those strategies are dominated by those involving only one account. Since defaulting with one account is not profitable on the equilibrium path, neither are any other strategies involving a second account. The reason for this result is that, whether one plans to default or not, focusing one's efforts on one account is the best strategy as this results in the attainment of the most reputation and therefore the greatest benefits.

We will therefore simplify the notation by dropping any notation referring to the second account's reputation as, according to the above proposition, it is always zero (for example, $V^b(n_1,0)=V^b(n_1)$).

Moreover, by combining (1), (2) and (7) and the fact that only one account is ever used, the DM-value function takes this following form:

$$V^b(n)=\sum_{j=0}^{\infty}\frac{(\beta(1-p)\sigma)^j}{(1-(1-\sigma)\beta(1-p))^{j+1}}\sigma(1-\theta)S_{n+j}, \tag{12}$$

where $S_n=u(q_n)-c(q_n)$ for all $n \leq N$ and $S_n=S_N$ for all $n>N$.

Lemma 1 *A pure-reputation equilibrium can be reduced to any sequence $\{q_n\}_{n=0}^{N}$ which satisfies (11) and (12).*

Proposition 2 *First best, $q^*=q_n$ for all n, cannot be a pure-reputation equilibrium.*

Proof. Suppose $q^*=q_n$ for all n is an equilibrium. Then by (12) it follows that $V(n)=V$ for all n. But then according to (11) $\omega(q^*)=0$ which is a contradiction. ■

This is not so surprising as trading first best for every level of reputation implies that reputation conveys no benefits and therefore the loss of reputation, the punishment for defaulting, has no bite. As a result, credit cannot be sustained.

Going forward, it useful to differentiate between two types of equilibria.

Definition 3 *A pure-reputation equilibrium is called not-too-tight if (11) holds with equality for all n. Otherwise, we call the equilibrium too-tight.*

The "not-too-tight" terminology originates from Alvarez and Jermann (2000). Equivalently, we could define it as saying $b_n = B_n$ for all n. That is, if an equilibrium is not-too-tight then it means that for each level of reputation, the buyer is indifferent between repaying and defaulting. Therefore, in each match, buyers issue the maximum amount of debt such that the buyer does not default. It is the most natural equilibrium to study as it maximizes the gains of trade for both buyers and sellers. Furthermore, observe that if there was no punishment for issuing more debt than the equilibrium amount, the bargaining solution would necessarily satisfy this constraint.[26]

3.4.1 Not-too-tight Equilibria

To understand the role that the punishment of issuing more debt than the equilibrium amount plays, we assume for the moment that there is none.[27] In that case, the terms of trade are determined solely by the debt limit B_n. However, we can immediately show that in that special case there is an unique equilibrium with no trade:

Proposition 3 *In a not-too-tight equilibrium it must be that $q_n = 0$ for all n.*

Again, for illustrative purposes we assume $\sigma = 1$ and $N = 1$.

Proof. Consider the following two ND-constraints for $n = 0$ and $n = 1$:

$$B_0 = \beta(1-p)[V^b(1) - V^b(0)] \geq c(q_0) \tag{13}$$

$$B_1 = \beta(1-p)[V^b(1) - V^b(0)] \geq c(q_1) \tag{14}$$

which implies that $B_1 = B_0$.

[26] This point holds more generally for other bargaining solutions aside the proportional solution we assumed here. Using the language of Hu et al. (2009), any mechanism that is coalition-proof maximises the amount of debt issued away from first best. Simply put, since agents cannot trade first best, they benefit from trading more. Therefore, there is a Pareto improvement by issuing as much debt as possible without default.

[27] Or alternatively that $\hat{b}_n \to \infty$ for all n.

Due to monotonicity, $q_1 \geq q_0$.[28] First, suppose $q^* = q_1 > q_0$. By (10), $c(q_1) < B_1$ and hence $c(q_0) < B_1 = B_0$. By (10) this implies $q_0 = q^*$. A contradiction since we assumed that $q_1 > q_0$. Second, suppose that $q^* > q_1 > q_0$. By (10), $b_1 = B_1$ and $b_0 = B_0$. But because $B_1 = B_0$ this implies $q_1 = q_0$ which is a contradiction with $q_1 > q_0$. Thus, $q_1 = q_0$. But then $V(1) = V(0)$ and $B_1 = B_0 = 0$. The only value of q_1 and q_0 that satisfy both (13) and (14) is $q_1 = q_0 = 0$. ■

A general proof can be found in the Appendix. This shows why it is necessary to not only punish buyers that default on their debt, but also those who issue too much (meaning exceeding the equilibrium amount of) debt. Absent this punishment, the only viable equilibrium is what we call not-too-tight which, as the proposition shows, implies that no positive consumption is incentive-feasible.

The intuition for this result can be understood in the following way: in the simplified case where $N = 1$ buyers either have reputation, $n = 1$, or they don't, $n = 0$. In this case, a buyer's propensity to default depends on the value of reputation, $V^b(1) - V^b(0)$, which is independent of his current level of reputation (see equations (13) and (14)). The seller has thus no incentive to treat a buyer with $n = 0$ different to a buyer with $n = 1$. But this is problematic because this makes reputation not valuable in the first place since the amount of trade for a buyer with $n = 0$ is the same as for a buyer with $n = 1$. If reputation has no value, there is no incentive to repay and sellers are thus not willing to accept any amount of debt.

We can also understand the result as the consequence of an externality. For an individual seller it would be best if other sellers extent less debt to to buyers with $n = 0$ below the debt limit such that $b_0 < B_0$ (so that the debt limit is "too-tight"). In that case, reputation becomes valuable and buyers would repay their promises for some positive amount of b. But given that other sellers behave this way, an individual seller optimally deviates and exhausts the debt limit fully, such that $b_0 = B_0$, and increases it's trade surplus. But given that all the sellers act like this, the debt limit falls to zero.

3.4.2 Too-Tight Equilibria

As we just seen, the only viable equilibria are too-tight equilibria. Thus, it must be that (11) is not holding with equality for at least one n. We proceed by studying under which conditions those equilibria exist and some properties that an optimal equilibrium must satisfy. To simplify some normative issues going forward, let us assume that $\theta \to 1$ which implies that seller's lifetime utility is always zero. In particular, it allows us to describe an optimal pure-reputation equilibrium, and some associated normative implications, in closed form.[29] We will say that a pure-reputation equilibrium is optimal if it maximizes the buyer's lifetime welfare.

[28] One can also prove this statement without invoking monotonicity. In that case one also has to check the possibility that $q_1 < q_0$. However, one can easily see that this is also in contradiction with the fact that $B_1 = B_0$.

[29] Alternatively, one could restrict attention to equilibra that maximize the buyer's welfare. We decided this approach was more appropriate as it allows us to make statements about welfare more generally and giving the buyer all the bargaining power is a common assumption in these types of models.

Formally, a pure-reputation equilibrium with $\theta \to 1$ is optimal if:

$$\max_{\{q_n\}_{n=0}^{N}} V^b(0) \text{ s.t. } V^b(\min(N, n+1)) - V^b(0) \geq \frac{c(q_n)}{\beta(1-p)} \ \forall n \text{ and (12).}$$

Proposition 4 *A sufficient condition for a pure-reputation equilibrium to be optimal is a)* $V^b(0) = \bar{V}(q_N)$ *and b)* $q_N = \hat{q}$ *where* $\hat{q}$ *and* $\bar{V}(q_N)$ *are respectively determined by*

$$\frac{u'(\hat{q})}{c'(\hat{q})} = 1 + \frac{1-\beta(1-p)}{\sigma\beta(1-p)} = 1 + \frac{r}{\sigma}, \tag{15}$$

$$\bar{V}(q_N) = \frac{\sigma[u(q_N) - c(q_N)]}{1-\beta(1-p)} - \frac{c(q_N)}{\beta(1-p)}. \tag{16}$$

Proof. Using the ND-constraint for $n = N$, inserting (12) for $n = N$ and solving for $V^b(0)$ yields $V^b(0) \leq \bar{V}(q_N)$ where $\bar{V}(q_N)$ is given by (16). The upper bound, $\bar{V}(q_N)$, depends solely on q_N. The first order condition of (16) with respect to q_N yields (15). If $V^b(0) = \bar{V}(q_N)$ then the equilibrium must be optimal by definition of the upper bound. ■

Hence, $\bar{V}(\hat{q})$ is the highest value of $V(0)$ that can be achieved. At this point it is not yet clear whether this upper bound can be attained throughout the entire parameter space, but assuming that it is achievable for a particular point in that space, then evidently we have found an optimal equilibrium at that point if a candidate equilibrium achieves that upper bound.

Let us now consider what kind of sequences may achieve $\bar{V}(\hat{q})$. In order to gain some intuition, it is useful to consider the simplified case where $N = 1$ and $\sigma = 1$ (again, accounts either have reputation or they do not). By using (12), the ND-constraints for $n = 0, 1$ simplify to:

$$V_1^b - V_0^b = S_1 - S_0 = \frac{c(q_1)}{\beta(1-p)} \tag{17}$$

and $q_0 \leq q_1$. Since optimality implies $q_1 = \hat{q}$, q_0 is set as such that (17) holds. If the implied q_0 is between zero and q_1 the equilibrium exists for some q_0. From (17) we can easily see that $q_0 \leq q_1$ is always satisfied as $S_1 > S_0$ requires $q_1 > q_0$. On the other hand, $q_0 > 0$ holds if and only if

$$u(\hat{q}) \geq c(\hat{q}) \frac{1+\beta(1-p)}{\beta(1-p)}. \tag{18}$$

These equations have a clear interpretation: equation (17) tells us that the value of having reputation, $V_1^b - V_0^b$, is given by higher surpluses which can be obtained with reputation, $S_1 - S_0 > 0$, and this value

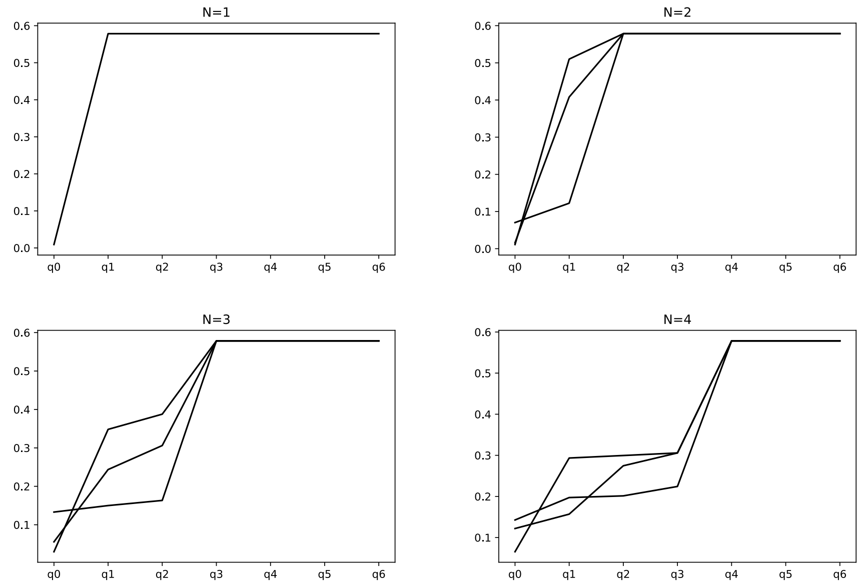

Figure 1: Different equilibria that achieve $\bar{V}(\hat{q})$

has to optimally equal the gain of defaulting, $\frac{c(q_1)}{\beta(1-p)}$.[30] Furthermore, even though $q_1 = q^*$ might be feasible, it is generally not optimal. The reason being that trading q^* would require a high level of reputation which in turn is only achievable if q_0 is sufficiently small. However, due to the concavity of $u(q)$ it is optimal to set $q_1 < q^*$ in order to increase q_0. Furthermore, according to (18), this is only a solution if $\beta(1-p)$ is sufficiently large. Intuitively, the highest value of reputation is attained if $q_0 = 0$. For some parameter values however, even this value is not sufficient to incentivize agents to repay $\hat{q}$.

In the more general case where $N > 1$ there exist several feasible and optimal sequences. This indeterminacy follows from the fact that in order to achieve the upper limit $\bar{V}(\hat{q})$ only two equations need to hold with equality: $q_N = \hat{q}$ and the ND-constraint for $n = N$. The other ND-constraints need not to hold with equality. Thus, there are $N+1$ variables to determine and 2 equations that pin them down. One way to find possible solutions is to compute them numerically. Figure 1 plots solutions with different N which all achieve $\bar{V}(\hat{q})$.[31]

There are however also closed form solutions. Consider the following: set $q_N = \hat{q}$ and $q_n \to 0$ for all $n < N-1$[32]. Notice, that the ND-constraint for all $n < N-1$ are in that case automatically satisfied. The

[30] Feasibility requires that the value of reputation weakly exceeds the gain from defaulting while optimality implies that it holds with equality. The reason is that a high value of reputation is costly.

[31] We assumed the following functional forms: $u(q) = \frac{q^{1-\eta}}{1-\eta}$ and $c(q) = q$. The parameters that were used are: $\beta = 0.96$, $\eta = 0.5$, $\sigma = 0.5$, $p = 0.1$.

[32] In order to gain reputation it must be that $q_n > 0$ for all n. The reason being that it is not observable whether an agent simply could not trade as he did not meet a seller or if he did meet one but could not trade since the reputational debt limit is zero. Hence, more accurately we should write $q_n \to 0$ for all $n < N-1$.

implied solution for q_{N-1} must however satisfy $0 \leq S_{N-1}$ (no negative surplus possible) and $S_{N-1} \leq S_N$ (otherwise the ND-constraint for $n = N-1$ would be violated). We can show the following:

Proposition 5 *There always exists a pure-reputation equilibrium such that $V^b(0) = \bar{V}(\hat{q})$ for a particular $N = \hat{N}$. A sufficient and necessary condition for a pure-reputation equilibrium to be optimal is $V^b(0) = \bar{V}(\hat{q})$ and $q_N = \hat{q}$.*

In the Appendix we proof Proposition (5) and derive $\hat{N}$ analytically. As we have seen in the special case where $N = 1$ and $\sigma = 1$, there was the possibility that the harshest possible punishment (setting $q_0 = 0$) was not sufficient to guarantee that agents repay their debt associated with trading $\hat{q}$. The proposition indicates that we can always increase N and thereby increase the scope for punishment in order to guarantee that an equilibrium always exists.

In conclusion, we have shown that a payment system can, in principle, be anonymous and be not reliant on money or any form of collateral. According to Proposition 5 such credit-based equilibria do always exist provided that N is chosen appropriately. However, while such credit-based equilibria always exist Proposition 4 indicates that they are costly in the sense that the punishment for default implicitly is imposed on young agents too. There is therefore a trade-off between making defaulting costly and letting young agents consume.

4 Closing Remarks

In this paper, we present a novel perspective on payment systems and anonymity, departing from the existing body of literature which either neglects agents' anonymity concerns entirely or imposes strict anonymity prerequisites that effectively rule out the existence of credit-based payment systems. Instead, our focus lies on pseudonymity, a particularly prevalent form of anonymity currently observed in many areas of the internet and, in particular, blockchains.

What can we learn from this exercise? First and foremost, there is often an assumption that anonymity and credit cannot coexist. Our analysis, from an economic standpoint, challenges this notion. While our examination primarily centers around credit-based payment systems, we believe our rationale extends to credit in a broader context. This holds particular significance for blockchain-related endeavors aiming to integrate credit within blockchain networks. Secondly, building upon the work of Kocherlakota (1998) who famously asserted that "money is memory", we concur with the idea that money serves as a rudimentary record-keeping technology. However, our research also highlights the importance of "money is anonymity", since there are also costs in maintaining anonymous credit-based systems.

Lastly, we wish to address some potential concerns arising from our framework. Firstly, as our study explores merely a subset of all possible credit equilibria, we refrain from making definitive assertions about the optimality of credit-based payment systems. However, we conjecture that our analysis did indeed capture the most efficient credit equilibrium. Secondly, a question arises regarding the significance of the assumption that buyers only engage with one seller per period. It is evident that if buyers were free to interact with an unlimited number of sellers, a dominant strategy would involve creating an infinite number of accounts, thereby reducing the debt limit to zero. While our current framework does not explicitly demonstrate this, we hypothesize that any cost of contacting additional sellers would suffice to prevent such a scenario. Thirdly, it is worth noting that our model simplifies many real-world complexities associated with credit. For example, we ignore potential heterogeneity in agents' ability to repay their debt. We leave all these considerations for future research.

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Appendix A.

Proof. Because $\alpha(n_1 \mid n_2) = 0$, seller's know that $\eta(b, n_1, 0) = 1$ if and only if $b \leq \min\{B(n_1, 0), \hat{b}_{n_1}\}$. Therefore, any $b > \min\{B(n_1, 0), \hat{b}_{n_1}\}$ cannot be the solution to the bargaining solution because it implies $q = 0$. We can therefore rewrite the bargaining problem as:

$$\begin{aligned}
&\max_{q_n, b_n} u(q_n) - (1-p)b_n \\
&\text{s.t. } (1-p)b_n = \theta u(q_n) + (1-\theta)c(q_n) \ \forall n \\
&b_n \leq \min\{B(n, 0), \hat{b}_n\}
\end{aligned}$$

The first order conditions with respect to q and b are respectively given by:

$$u'(q) - \lambda[\theta c'(q) + (1-\theta)u'(q)] = 0 \tag{19}$$

$$-(1-p) + \lambda(1-p) - \mu = 0, \tag{20}$$

where λ and μ are the Lagrangian multiplier with respect to the first and second constraint respectively. First, if the buyer is unconstrained then $q = q^*$ and $b_n^* = \frac{\theta u(q^*) + (1-\theta)c(q^*)}{(1-p)}$. The buyer is therefore only unconstrained if $b_n^* \leq \min\{B(n, 0), \hat{b}_n\}$. Second, if the buyer is constrained, i.e. $b_n^* > \min\{B(n, 0), \hat{b}_n\}$, then $b_n = \min\{B(n, 0), \hat{b}_n\}$ and q_n solves

$$(1-p)\min\{B(n, 0), \hat{b}_n\} = \theta u(q_n) + (1-\theta)c(q_n) \ \forall n.$$

Since the left-hand side is a constant, non-negative number given n and the right-hand side is a monotonically increasing function of q with minimum zero when $q = 0$, it follows that there exists a unique q that solves (21). ■

Appendix B.

Proof. We aim to demonstrate the satisfaction of (8) for all values of n. In other words, we want to prove that it is never advantageous for a buyer to open up and gain reputation on a second account. We employ a proof by contradiction and assume that the buyer opens a second account at time t_0, while already possessing a first account with a reputation of n_1. For brevity, we denote $\tilde{\beta} \equiv \beta(1-p)$. There are two primary scenarios concerning the reputation on the first account at the point in time where the buyer opens the second account:

1. There exists an $n < n_1$ such that $q_{n_1} > q_n$. There are four feasible strategies which cover all possibilities:

   (a) Consider the case where the buyer opens a second account and accumulates reputation until it reaches $n_2 = n_1 + 1$. At this point, the buyer's state variables can be expressed as $(n_1, n_1 + 1)$, assuming without loss of generality that this occurs in period t_1. The *deviation value* can be formulated as follows:

   $$\tilde{V} = \mathcal{S} + \beta^{t_1 - t_0 - 1}[u(q_{n_2 - 1}) - c(q_{n_2 - 1})] + \beta^{t_1 - t_0} V(n_1, n_1 + 1) \tag{21}$$

   where $\mathcal{S}$ is the discounted pay-off of using the second account between t_0 and $t_1 - 1$.[33] In t_0 there is another feasible strategy: alternatively, the agent could use the first account one more time in t_0 and then start using the second account so that in t_1 the agent's state variables are $(n_1 + 1, n_1)$. This *alternative value* is then given by:

   $$V^* = [u(q_{n_1}) - c(q_{n_1})] + \beta\mathcal{S} + \beta^{t_1 - t_0} V(n_1 + 1, n_1). \tag{22}$$

[33] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{n_2 - 2} \beta^{\hat{t}_n - t_0}[u(q_n) - c(q_n)]$ where $\hat{t}_n$ is the time where the buyer trades with the second account with reputation n.

The deviation value can only be optimal if it weakly exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values and recognizing that $q_{n_1} = q_{n_2-1}$ and $V(n_1+1, n_1) = V(n_1, n_1+1)$:

$$[u(q_{n_1}) - c(q_{n_1})](1 - \beta^{t_1-t_0-1}) \leq \mathcal{S}(1-\beta). \tag{23}$$

Since there exists an $n_2 \leq n_1$ such that $q_{n_1} > q_{n_2}$ it must be that $\mathcal{S} < [u(q_{n_1}) - c(q_{n_1})] \sum_{j=0}^{t_1-t_0-1} \beta^j = [u(q_{n_1}) - c(q_{n_1})] \frac{1-\beta^{t_1-t_0}}{1-\beta}$ and therefore the previous inequality implies: $u(q_{n_1}) - c(q_{n_1}) < u(q_{n_1}) - c(q_{n_1})$. A contradiction.

(b) Suppose the buyer opens a second account and builds up reputation until $n_2 \leq n_1$. He then switches accounts, borrows and repays with the first account so that the agent's state variables are (n_1+1, n_2) at this point. Without loss of generality, let us suppose the switch occurs in period t_1. The deviation value can be written as:

$$\tilde{V} = \mathcal{S} + \beta^{t_1-t_0}[u(q_{n_1}) - c(q_{n_1})] + \beta^{t_1-t_0+1} V(n_1+1, n_2) \tag{24}$$

where $\mathcal{S}$ is the discounted pay-off of using the second between t_0 and $t_1 - 1$.[34] In t_0 there is another feasible strategy: alternatively, the agent could use the first account one more time in t_0 and then start using the second account so that in t_1 the buyer's state variables are the same in $t_1 + 1$. This alternative value is then given by:

$$V^* = [u(q_{n_1}) - c(q_{n_1})] + \beta\mathcal{S} + \beta^{t_1-t_0+1} V(n_1+1, n_2). \tag{25}$$

The deviation value can only be strictly beneficial if it's exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values yields:

$$[u(q_{n_1}) - c(q_{n_1})](1 - \beta^{t_1-t_0}) < \mathcal{S}(1-\beta). \tag{26}$$

Since there exists an $n_2 \leq n_1$ such that $q_{n_1} > q_{n_2}$ it must be that $\mathcal{S} < [u(q_{n_1}) - c(q_{n_1})] \frac{1-\beta^{t_1-t_0-1}}{1-\beta}$ and therefore the previous inequality implies: $u(q_{n_1}) - c(q_{n_1}) < u(q_{n_1}) - c(q_{n_1})$. A contradiction.

(c) Suppose the buyer opens a second account and builds up reputation until $0 \leq n_2 \leq n_1$. At some point, the agent defaults on the second account, and without loss of generality, let us assume this default occurs in period t_1:

$$\tilde{V} = \mathcal{S} + \beta^{t_1-t_0} u(q_{n_2}) + \beta^{t_1-t_0+1} \tilde{V}. \tag{27}$$

Importantly, the buyer's state variables and therefore the continuation value are the same before using the second account and defaulting on the second account. However, this would imply that the first account would never be used again and $\tilde{V} = V(n_1, 0)$. But since $\tilde{V} = V(n_1, 0)$ is a possible strategy with state variables $(0,0)$, it must be that $V(0,0) \geq V(n_1, 0) = \tilde{V}$. But according to the ND-constraint (11) this implies $q_n = 0$ for all $n < n_1$ and therefore $V(0,0) < V(n_1, 0) = \tilde{V}$. A contradiction.

(d) Suppose the buyer opens a second account and builds up reputation until $n_2 \leq n_1$. Then the agent switches back to the first account, borrows from it, and immediately defaults on that account. Without loss of generality, let us suppose the switch and default occur in period t_1. The deviation value can then be written as:

$$\tilde{V} = \mathcal{S} + \beta^{t_1-t_0} u(q_{n_1}) + \beta^{t_1-t_0+1} V(0, n_2). \tag{28}$$

Observe, that in t_0 there is another feasible strategy: instead the agent defaults in t_0 on the first account and then starts using the second account so that in t_1 the agent's state variables are the

[34] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{n_2-1} \beta^{\hat{t}_n - t_0}[u(q_n) - c(q_n)]$ where $\hat{t}_n$ is the time where the buyer trades with the second account with reputation n.

same as in t_1+1. The alternative value is then given by:

$$V^* = u(q_{n_1}) + \beta\mathcal{S} + \beta^{t_1-t_0+1}V(0,n_2). \tag{29}$$

The deviation value can only be strictly beneficial if it weakly exceeds the alternative value: $V^* \leq \tilde{V}$. Inserting the values yields:

$$u(q_{n_1})(1-\beta^{t_1-t_0-1}) < \mathcal{S}(1-\beta). \tag{30}$$

Since there exists an $n_2 \leq n_1$ such that $q_{n_1} > q_{n_2}$ it must be that $\mathcal{S} \leq [u(q_{n_1}) - c(q_{n_1})]\frac{1-\beta^{t_1-t_0-1}}{1-\beta}$ and therefore the previous inequality implies: $0 < -c(q_{n_1})$. A contradiction.

2. There exists no $n < n_1$ such that $q_{n_1} > q_n$. Let us therefore denote $\hat{n}$ the lowest level of reputation such that $q_{\hat{n}} > q_n$ for all $n < \hat{n}$.

   (a) The buyer trades with both accounts until the reputation of both accounts is given by $(\hat{n}_1, \hat{n}_2)$ where $\hat{n}_1 < \hat{n}$ and $\hat{n}_2 < \hat{n}$. The buyer then defaults on either of these accounts. Without loss of generality, let us suppose the default occurs on the second account and the switch and default occur in period t_1. The deviation payoff can be written as:

   $$\tilde{V} = \mathcal{S} + \beta^{t_1-t_0}u(q_{n_2}) + \beta^{t_1-t_0+1}V(\hat{n}_1, 0) \tag{31}$$

   where $\mathcal{S}$ is the discounted pay-off off using the first and second account between t_0 and t_1-1.[35] In t_0 there is another feasible strategy: alternatively, the agent could default on the second account with reputation zero instead of trading with the account. This alternative value is then given by:

   $$V^* = \mathcal{S}^* + \beta^{t_1-t_0}u(q_{n_2}) + \beta^{t_1-t_0+1}V(\hat{n}_1, 0) \tag{32}$$

   where $\mathcal{S}^*$ is the discounted payoff of defaulting on the second account between t_1 and t_0.[36] Since $q_n = q_{n_1} = q_{n_2}$ for all $n < n_1$ and $n < n_2$, this implies $\mathcal{S}^* > \mathcal{S}$ if $q_{n_1} = q_{n_2} > 0$. A contradiction.

   (b) The buyer does not default before either $n_1 = \hat{n}$ or $n_2 = \hat{n}$ is reached in period t_1. Without loss of generality, let us suppose $n_1 = \hat{n}$. It is easy to verify that we can now apply exactly the same argument as in Case 1 and conclude that the buyer has no incentive to increase the reputation on the account with lower levels of reputation.[37] If the agent will not use the second account to accumulate further reputation then he either defaults on the account or he will never use it again:

      i. The buyer defaults on the second account. However, according to the same argument as in sub-case a), there is a better strategy where the buyer always defaults on the second account instead of accumulating reputation. A contradiction.

      ii. The buyer never uses the second account again. The deviation value is then given by:

      $$\hat{V} = \mathcal{S} + \beta^{t_1-t_0}V(\hat{n}_1, \hat{n}_2) \tag{33}$$

      where $\mathcal{S} = [u(q_0) - c(q_0)]\frac{1-\beta^{t_1-t_0}}{1-\beta}$ is the discounted payoff of using the second and first account between t_1 and t_0. In t_0 there is another feasible strategy: instead of using the second account, the buyer could use only the first account. Let us denote the time when the buyer achieves $n_1 = \hat{n}$ as $t_1^* < t_1$. The alternative value is then given by:

      $$V^* = \mathcal{S}^* + \beta^{t_1^*-t_0}V(\hat{n}_1, 0) \tag{34}$$

[35] The explicit expression is given by $\mathcal{S} = \sum_{n=0}^{\hat{n}_1-n_1+n_2} \beta^{\hat{t}_n-t_0}[u(q_0) - c(q_0)]$ where $\hat{t}_n$ is the time where the buyer trades with the second account with reputation n.

[36] The explicit expression is given by $\mathcal{S}^* = \sum_{n=0}^{\hat{n}_1-n_1+n_2} \beta^{\hat{t}_n-t_0}u(q_0)$ where $\hat{t}_n$ is the time where the buyer trades with the second account with reputation n.

[37] Essentially, the argument in Case 1 is independent of the level of reputation on the second account as long as the second account has lower reputation than the first.

where $\mathcal{S}^* = [u(q_0) - c(q_0)]\frac{1-\beta^{t_1^*-t_0}}{1-\beta}$ is the discounted payoff of using the first account between t_1^* and t_0. The deviation value can only be strictly beneficial if it exceeds the alternative value: $V^* \leq \hat{V}$:

$$\frac{u(q_0) - c(q_0)}{1-\beta} \geq V(\hat{n}_1, 0) \tag{35}$$

where we used the fact that $V(\hat{n}_1, 0) = V(\hat{n}_1, \hat{n}_2)$ since the second account will never be used from time t_1 onwards. This implies that the deviation value is bounded from above. But then there exists a second alternative strategy where the agent always defaults on the first account from t_1 onwards yielding:

$$V^{**} = \frac{u(q_0)}{1-\beta} > \frac{u(q_0) - c(q_0)}{1-\beta} \geq \hat{V}. \tag{36}$$

A contradiction.

We conclude that opening a second account cannot be optimal. ■

Appendix C.

Proof. Consider the following two ND-constraints for $n = N$ and $n = N-1$:

$$B_{N-1} = \beta(1-p)[V^b(N) - V^b(0)] \geq c(q_{N-1}) \tag{37}$$

$$B_N = \beta(1-p)[V^b(N) - V^b(0)] \geq c(q_N) \tag{38}$$

which implies that $B_N = B_{N-1}$.

Due to monotonicity, $q_N \geq q_{N-1}$.[38] First suppose $q^* = q_N > q_{N-1}$. By (10), $c(q_N) \leq B_N$ and hence $c(q_{N-1}) < B_N = B_{N-1}$. By (10) this implies $q_{N-1} = q^*$. A contradiction since we assumed that $q_N > q_{N-1}$. Second, suppose that $q^* > q_N > q_{N-1}$. By (10), $b_N = B_N$ and $b_{N-1} = B_{N-1}$. But because $B_N = B_{N-1}$ this implies $q_N = q_{N-1}$ which is a contradiction with $q_N > q_{N-1}$. Thus, $q_N = q_{N-1}$.

Now consider, q_{N-1} and q_{N-2}. One can see from equation (12) that $q_N = q_{N-1}$ implies $V(N) = V(N-1)$. But then by the same argument as above, it must be the case that $B_{N-1} = B_{N-2}$ and thus $q_{N-1} = q_{N-2}$. Applying this argument recursively to all q_n implies that $q_n = q$ for all n. But then $V(n) = V(0)$ for all n and thus $B_n = 0$ for all n. ■

Appendix D.

Consider the following sequence: $q_N = \hat{q}$ and $q_n \to 0$ for all $n < N-1$. The ND-constraints for $n < N-1$ are thus satisfied. The ND-constraint for $n = N-1$ is satisfied if $q_{N-1} < q_N$. q_{N-1} is determined such that the ND-constraint for $n = N$ is satisfied:

$$\beta(1-p)[V^b(N) - V^b(0)] = c(q_N).$$

We insert (12). It follows that:

$$\frac{(1+r)}{r}\sigma S_N - \left[\left[\frac{\sigma}{r+\sigma}\right]^{N-1}\frac{(1+r)}{(r+\sigma)}\sigma S_{N-1} + \left[\frac{\sigma}{r+\sigma}\right]^{N}\frac{(1+r)}{r}\sigma S_N\right] = (1+r)c(q_N).$$

We solve the term for S_{N-1}:

[38] One can also prove the statement without the assumption of monotonicity. In that case one also has to check the possibility that $q_N < q_{N-1}$. However, one can easily see that this is also in contradiction with the fact that $B_N = B_{N-1}$.

$$S_{N-1} = \left[\left[\frac{r+\sigma}{\sigma}\right]^N - 1\right]\frac{\sigma}{r}S_N - \left[\frac{r+\sigma}{\sigma}\right]^N c(q_N)$$

We know that $S_{N-1} = u(q_{N-1}) - c(q_{N-1})$ and therefore q_{N-1} is determined. For this to be a solution it needs to be that $0 \leq q_{N-1} < q_N$. We show that this is only fullfilled for one specific N. We first derive the set of N under which $q_{N-1} < q_N$. This is the case if $S_N > S_{N-1}$. We insert our solultion for S_{N-1} derived above. This yields

$$S_N > \left[\left[\frac{r+\sigma}{\sigma}\right]^N - 1\right]\frac{\sigma}{r}S_N - \left[\frac{r+\sigma}{\sigma}\right]^N c(q_N)$$

respectively,

$$\frac{ln\left[\frac{\frac{r+\sigma}{r}}{\frac{\sigma}{r} - \frac{c(q_N)}{S_N}}\right]}{ln\left[\frac{r+\sigma}{\sigma}\right]} > N$$

In a second step we derive the set of N under which $0 \leq q_{N-1}$. This is the case if $0 \leq S_{N-1}$. Given our solution for S_{N-1}:

$$\left[\left[\frac{r+\sigma}{\sigma}\right]^N - 1\right]\frac{\sigma}{r}S_N - \left[\frac{r+\sigma}{\sigma}\right]^N c(q_N) \geq 0$$

respectively,

$$N \geq \frac{ln\left[\frac{\frac{\sigma}{r}}{\frac{\sigma}{r} - \frac{c(q_N)}{S_N}}\right]}{ln\left[\frac{r+\sigma}{\sigma}\right]}.$$

The difference between the upper and the lower bound is 1. Given that N is an integer value this means that the solution can only be implemented if

$$N = \left\lceil \left(ln\left(\frac{\sigma}{r}\right) - ln\left(\frac{\sigma}{r} - \frac{c(q_N)}{S_N}\right)\right)\frac{1}{ln\left(\frac{r+\sigma}{\sigma}\right)}\right\rceil.$$

A necessary and sufficient condition for existence is $\frac{\sigma}{r} - \frac{c(q_N)}{S_N} \geq 0$. From rearranging (16) it follows that $\bar{V}(q_N) = (1+r)S(N)\Big(\frac{\sigma}{r} - \frac{c(q_N)}{S(N)}\Big)$. Given we maximize $\bar{V}(q_N)$ in our optimal equilibrium and it is always possible to set $\bar{V}(q_N) = 0$ if this would be the value that maxmizes the function, we know that $\bar{V}(q_N) \geq 0$.