

# Transaction Costs, the Value of Convenience, and the Cross-Section of Safe Asset Returns\*

Ragnar Juelsrud<sup>†</sup>

Plamen Nenov<sup>‡</sup>

Fabienne Schneider<sup>§</sup>

Olav Syrstad<sup>¶</sup>

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## Abstract

We study the cross-section of safe asset returns in a model with a micro-founded demand for liquidity and heterogeneous asset transaction costs. Central to the model is the “value of convenience”, an endogenous marginal value of liquidity that measures the equilibrium degree of liquidity risk-sharing. Changes in asset supply, payoff risk or transaction costs can affect the returns on all assets via the value of convenience. Heterogeneous transaction costs give rise to a pecuniary externality that investors fail to internalize when forming portfolios. We document new facts about US Treasuries that support the main predictions of our theory.

**Keywords:** pecuniary externality, convenience yield, Treasury-OIS spread, quantitative easing

**JEL Codes:** G12, E44

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<sup>†</sup>University of Oslo and CEPR, e-mail: ragnar@juelsrud.no

<sup>‡</sup>Norges Bank, e-mail: plamen.nenov@norges-bank.no

<sup>§</sup>Bank of Canada, e-mail: fschneider@bank-banque-canada.ca

<sup>¶</sup>BI Norwegian Business School, e-mail: olav.syrstad@bi.no

# 1 Introduction

Safe assets, such as government bonds, are not just risk-free assets. Many investors hold these assets for the liquidity services they provide, since they either have a short enough maturity to match investors' liquidity needs or can be readily converted to cash in liquid secondary markets with little price impact. Therefore, safe assets tend to command an additional premium, often referred to as a liquidity premium or convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012). However, it is well-known that not all safe assets are the same, and that there exists substantial variation in safe assets' convenience yields.<sup>1</sup>

In this paper we study the pricing and welfare implications of safe asset heterogeneity, using a highly tractable theory with multiple assets, agent heterogeneity, an endogenous marginal value of liquidity, and aggregate risk. We use the model to illustrate a pecuniary externality that arises from the interaction of heterogeneous safe asset transaction costs and liquidity demand and analyze different welfare-improving policies.

In our model, agents with recursive Epstein and Zin (1989) preferences face uninsurable idiosyncratic liquidity risk modeled via interim impatience shocks. This idiosyncratic risk creates a demand for liquidity. Specifically, agents self-insure against the risk of needing to consume early by holding and trading assets. Assets differ in their exposure to an aggregate payoff risk factor, their transaction costs when traded, and their terminal payoffs. Transaction costs are assumed to be proportional to the value of the assets sold. We refer to the transaction cost on an asset as that asset's illiquidity. A larger transaction cost incurred during a sale implies greater illiquidity. We assume there are multiple risk-free assets that differ in terms of their transaction costs, terminal payoffs and relative supply, as well as a single illiquid risky asset (a market portfolio). We follow

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<sup>1</sup>See, for example, the term structure of relative (in)convenience in maturity-matched Treasury-OIS positions that we document in Section 5.

Acharya and Pedersen (2005) and other papers in the literature on asset pricing with transaction costs and treat per-unit transaction costs as asset-specific primitives, consistent with persistent liquidity differences across otherwise very similar securities, such as the on-the-run/off-the-run Treasury premium (Krishnamurthy, 2002). Similar to Acharya and Pedersen (2005) we interpret these transaction costs much more broadly than simple fees or bid-ask spreads and instead let them encompass market liquidity considerations related to price impact and trading delays.

The intensity of idiosyncratic shocks that agents are exposed to, combined with *the entire* asset transaction-cost and relative-supply structure, as well as the curvature in the period utility associated with a finite elasticity of intertemporal substitution, determine the equilibrium liquidity scarcity in this economy, which is fully summarized by a single equilibrium object. We refer to this important equilibrium object as *the value of convenience*. The value of convenience is defined as the ratio of equilibrium marginal utilities of consumption of an impatient versus a patient investor, and as such it reflects the degree of liquidity risk-sharing that is attained in equilibrium. Intuitively, a higher value of convenience implies a lower level of liquidity risk-sharing between investors. Transaction costs play a key role in determining the value of convenience since the total transaction cost expenditure incurred by asset sellers creates a wedge between the marginal utilities of patient and impatient investors. With proportional transaction costs, such as bid-ask spreads and price impact costs, asset prices also impact this wedge, giving rise to a pecuniary externality that investors fail to internalize when making their portfolio decisions.

To examine the cross-section of safe asset returns, we characterize *relative* inconvenience yields, defined as a return on an asset relative to the most liquid (zero transaction cost) safe asset in the economy. This relative inconvenience yield depends not only on an asset's own illiquidity but also on the value of convenience. A higher value of convenience

implies higher relative inconvenience yields and a wider cross-section of safe asset returns. Therefore, the value of convenience acts as a pricing factor for assets in the economy.

We use our framework to analyze how a change in asset supply or in the illiquidity of *a single* asset affects the returns on *all* assets via general equilibrium effects operating through the value of convenience. For example, substituting less liquid with more liquid safe assets one-for-one purifies the pool of safe assets, decreases the value of convenience, and compresses relative inconvenience yields. We also show that when safe assets are on average more liquid than the risky market portfolio – a natural assumption – an increase in aggregate risk or risk aversion *decreases* the value of convenience, narrowing the relative inconvenience yields across safe assets. The reason for this surprising outcome is a revaluation effect caused by a “flight to safety” adjustment in agents’ portfolios. Intuitively, *ceteris paribus*, higher equilibrium valuations of the relatively more liquid risk-free safe assets (and, respectively, lower valuations of the less liquid risky assets) after a “flight to safety” episode shrink the transaction cost expenditure of impatient investors and lower the value of convenience.

Two clarifications sharpen this seemingly counter-factual prediction. First, it concerns the relative convenience yields on *risk-free* assets. Indeed, spreads measured against risky benchmarks, such as the AAA-Treasury spread, could widen with payoff risk because the risky leg reprices, so crisis spikes in those measures partly reflect risk premia rather than liquidity scarcity. In our empirical exercises we therefore measure convenience using duration-matched positions in safe instruments – see below. Second, crises may independently raise the demand for liquidity itself. Specifically, we show that when the incidence of liquidity shocks rises with payoff risk, an offsetting force widens relative inconvenience yields, with the net effect governed by the strength of that liquidity-demand response. In our empirical analysis we hold this channel fixed through our controls, so that the VIX coefficient isolates the revaluation effect – and, consistent with the theory, it is negative.

In addition to serving as a pricing factor, the value of convenience fully summarizes the welfare consequences of imperfect liquidity-risk sharing. The higher the value of convenience, the lower the equilibrium welfare. In our framework there is a pecuniary externality that matters for aggregate welfare. Specifically, price-taking investors do not internalize how their asset demand affects asset prices and ultimately the equilibrium wedge between patient and impatient investors' marginal utilities. We show in a simple two-asset version of our model that distorting the asset demand of investors towards the more liquid asset and away from the less liquid asset improves social welfare. Intuitively, ex post patient investors, who are not affected by the transaction costs, treat all assets as perfect substitutes. However, from a social optimum perspective, the more liquid asset is more valuable because it carries a lower transaction cost. Therefore, the distortion in asset demand brings the private and social value of the two assets closer together and increases aggregate welfare. Consequently, there are welfare-improving policy interventions. For example, building directly on the two-asset intuition above, a revenue-neutral set of taxes and subsidies that distort safe asset after-tax payoffs by increasing the relative after-tax payoff on the most liquid asset increases aggregate welfare. An alternative welfare-improving policy involves substituting illiquid for liquid assets. Through the lens of our simple theory, such policies broadly resemble central bank balance sheet policies as well as government debt management policies.

Finally, we show a new set of stylized facts that are consistent with the predictions of a version of our model that explicitly links liquidation costs with asset maturity. In the data we look at the difference between a Treasury-OIS spread of a given maturity (1 year to 10 years) and the 3-month Treasury-OIS spread. First we show that there is a term structure in these relative spreads. In addition, whenever Treasury-OIS relative spreads increase, they increase more for longer maturities. Since, institutionally, a Treasury-OIS spread with a higher tenor has a higher liquidation cost, these findings are directly

consistent with our theory. We also show that Treasury-OIS relative spreads co-move positively with the MOVE Index, which measures yield volatility of US Treasuries and is thus closely correlated with illiquidity and transaction costs in the Treasury market (Duffie et al., 2023). However, they co-move *negatively* with the VIX Index, consistent with revaluation effects of safe assets during “flight to safety” episodes.

**Related literature** Our paper contributes to the large and growing literature on safe assets and convenience yields, spurred by the seminal contribution of Krishnamurthy and Vissing-Jorgensen (2012) (see Gorton et al. (2012), Caballero and Farhi (2013), Krishnamurthy and Vissing-Jorgensen (2015), Sunderam (2015), Caballero et al. (2016), Nagel (2016), Azzimonti and Yared (2019), He et al. (2019), Gorton (2020), Ahnert and Macchiavelli (2021), Christensen and Mirkov (2021), Diamond and Van Tassel (2021), Jiang et al. (2021), Kacperczyk et al. (2021), Acharya and Dogra (2022), Barro et al. (2022), Brunnermeier et al. (2024), Gorton and Ordóñez (2022), Van Binsbergen et al. (2022), Engel and Wu (2023), Krishnamurthy and Li (2023), Stein and Wallen (2023), Lenel (2023), Herrenbrueck and Wang (2025), Mota (2023), Acharya and Laarits (2025), among others).<sup>2</sup> Following Krishnamurthy and Vissing-Jorgensen (2012), much of this literature models convenience demand via a reduced-form money-in-the-utility function. Our contribution to this literature is to introduce a micro-founded general equilibrium model in which convenience yields and the marginal value of liquidity arise endogenously from liquidity risk-sharing in the presence of transaction costs. The micro-foundations of liquidity demand also allow us to analyze welfare and the welfare effects of policy interventions.

Our emphasis on transaction costs brings our paper close to models of equilibrium asset prices with transaction costs and trading frictions (Amihud and Mendelson (1986),

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<sup>2</sup>See Brunnermeier et al. (2024), Caballero et al. (2017), Golec and Perotti (2017), and Gorton (2017) for reviews of the literature.

Aiyagari and Gertler (1991), Heaton and Lucas (1996), Vayanos (1998), Vayanos and Vila (1999), Holmström and Tirole (2001), Huang (2003), Lo et al. (2004), Vayanos (2004), Acharya and Pedersen (2005), Duffie et al. (2005)).<sup>3</sup> As in our framework, in these models, transaction costs imply a liquidity premium for assets with different transaction costs or trading frictions.<sup>4</sup> Our contribution relative to much of that literature is to consider an environment in which transaction cost expenditures impact the equilibrium marginal value of liquidity rather than holding that equilibrium marginal value fixed. This endogenous “value of convenience” is common across assets and causes the liquidity characteristics, supply, and price of any one asset to affect the convenience yields of all other assets. Closest to this mechanism, Vayanos and Vila (1999) analyze a model with two risk-free assets, one without a transaction cost and one with a positive transaction cost. They consider an OLG framework with 3-period-lived agents that trade the assets for life-cycle reasons, with agents at different stages of their life-cycles demanding different types of assets. Relative to their model, we propose a framework where liquidity demand and pricing arise from the combination of uninsured idiosyncratic liquidity shocks and transaction costs. Our framework can also accommodate multiple risk-free assets in addition to a risky asset (the market portfolio).

Most closely related to our paper, Brunnermeier et al. (2024) also generate the demand for safe assets from uninsurable idiosyncratic risk and price an equilibrium flow of service benefits – their retrading perspective on convenience yields – which is conceptually related to the value of convenience in our framework. Where the two papers complement one another is in the asset space: they study a single safe asset, while we study a cross-section

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<sup>3</sup>See Amihud et al. (2006) and Vayanos and Wang (2013) for reviews of the literature. See also Constantinides (1986), Duffie and Sun (1990), Davis and Norman (1990), Grossman and Laroque (1990), and Dumas and Luciano (1991) for partial equilibrium portfolio allocation models in the presence of transaction costs.

<sup>4</sup>In addition to differences in transaction costs or liquidity across assets, the liquidity premium may arise from liquidity risk. We also provide a micro-foundation for the transaction costs arising from a specific form of liquidity risk – a correlation between the aggregate liquidity conditions in the economy and an asset’s returns, similar to Holmström and Tirole (2001) and Acharya and Pedersen (2005).

of many assets with heterogeneous transaction costs. It is precisely this heterogeneity in transaction costs that delivers our main theoretical insights, including the pecuniary externality we identify, and which is absent in a single asset framework.

The central role of transaction cost expenditures, which are endogenously determined in equilibrium, for aggregate liquidity conditions also brings our theory close to models of endogenous liquidity, such as in Vayanos and Wang (2012), Kurlat (2013), Malherbe (2014), and Bigio (2015) and more recently in Coppola et al. (2026). Unlike these models, which endogenize per-unit transaction costs via asymmetric information frictions, or thick-market externalities in search, we assume that the per-unit transaction costs are exogenous and instead focus on the *endogenous* pricing of assets with different transaction costs when the marginal value of liquidity is also endogenous.

Finally, our empirical measurement of convenience yields follows a recent wave of literature applying the OIS rate as a proxy for the risk-free rate and the corresponding Treasury-OIS spread as a proxy for the convenience yield embedded in Treasuries (Filipovic and Trolle (2013), He et al. (2022), Klingler and Sundaresan (2023), Du et al. (2023), Fleckenstein and Longstaff (2024), among others). Du et al. (2023) and Klingler and Sundaresan (2023) relate the increase in the Treasury-OIS spread since the Global Financial Crisis (GFC) to the increase in the supply of Treasuries.<sup>5</sup> Specifically, Du et al. (2023) argue that Treasury supply is a significant driver of the Treasury-OIS spreads and as a result contributed to the regime shift where dealers have positive net positions in Treasuries. In contrast to these articles, our primary empirical interest lies in comparing Treasury-OIS spreads across tenors, and we are agnostic about whether the (in)convenience arises primarily in the Treasury or OIS arm of the position. Our analysis, therefore, complements recent work by Mota (2023), which also focuses on understanding relative convenience yields, albeit in the context of US corporate bonds.

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<sup>5</sup>In an earlier contribution, Greenwood and Vayanos (2014) also analyze pricing effects of relative supply of Treasuries.

## 2 Theory

We start by informally discussing the main features of our model. Agents in our model have standard Epstein and Zin (1989) recursive preferences and form portfolios over risk-free and risky assets at an initial period.<sup>6</sup> They may then have interim trading needs, which arise because of uninsurable idiosyncratic shocks, as in the asset-pricing with uninsured idiosyncratic risk literature (e.g., Bewley (1979), Aiyagari and Gertler (1991), Constantinides and Duffie (1996), Heaton and Lucas (1996), Heaton and Lucas (2000), Constantinides (2002), Di Tella (2020), Brunnermeier et al. (2024)). In our framework, these idiosyncratic shocks will take the form of shocks to marginal utility, making the agent more impatient to consume in the period. Assets in our framework will each have a proportional transaction cost that is borne by the seller of the asset in the interim period. As in Acharya and Pedersen (2005), we interpret this transaction cost much more broadly than simple fees or bid-ask spreads and instead let this cost encompass liquidity considerations related to price impact and trading delays. In Section 5.1 we provide a micro-founded example of our model, in which transaction costs increase with asset maturity and also incorporate liquidity risk – the covariance between the price of an asset and the aggregate liquidity needs in the economy. Finally, we assume that there is always a risk-free asset with zero transaction cost. This is consistent with theories of security design, in which a riskless security has the lowest transaction cost (see Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), Dang et al. (2009)).<sup>7</sup>

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<sup>6</sup>Using Epstein-Zin preferences allows for a separation of the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion. In our setting this allows us to separately consider the effects of changes in payoff risk or risk aversion on asset prices and relative convenience yields, holding the EIS fixed at unity.

<sup>7</sup>Note that our framework only models the demand for convenience via liquidity needs, and we do not explicitly model any additional demand for safety (e.g., due to limited participation in markets for risky assets (Vissing-Jorgensen, 2002) or because of special properties of using safe and informationally insensitive assets as collateral (Dang et al., 2010)). Our model can easily be modified to accommodate the latter channel, as borrowing against collateral for liquidity purposes (i.e., funding liquidity) and liquidating an asset (i.e., market liquidity) are closely related concepts.

## 2.1 Model set-up

The model has 3 periods:  $t = 0, 1, 2$ . The state at  $t = 2$  is uncertain and described by the realization  $z$  of a continuous random variable  $Z$ . There is a unit measure of 3-period lived investors. Investors have non-storable endowments  $Y_0$  at  $t = 0$  and  $Y_1$  at  $t = 1$ . A measure  $\lambda$  of investors are subject to a liquidity/impatience shock at the beginning of  $t = 1$ . They are denoted by type  $s = \bar{S}$  and are referred to as impatient. The remaining investors are denoted by  $s = \underline{S}$  and are referred to as patient. Investors consume in all three periods,  $c_0, c_{1,s}, c_{2,s}$ , for  $s \in \{\underline{S}, \bar{S}\}$ . The liquidity shock makes investors value consumption at  $t = 1$  more and not value consumption at  $t = 2$  anymore.

There are two types of assets. The first type is risk-free assets, and there are  $N$  such assets. We denote the set of these assets by  $\mathbb{I} = \{1, 2, \dots, N\}$ . The second type is risky assets, and for simplicity we assume there is only one such asset, which we refer to as the market portfolio and denote it by  $m$ .

Risk-free asset  $i$  has a constant  $t = 2$  payoff of  $\delta^i$ . For the market portfolio, we denote the payoff by  $\varphi^m(z)$ . We further assume that  $\log \varphi^m(z)$  is distributed according to  $\log \varphi^m(z) \sim N(\mu - \sigma^2/2, \sigma^2)$ .<sup>8</sup> The total supply of all risk-free assets is denoted by  $Q^f$ , while for the market portfolio the total supply is  $Q^m$ . The supply of each risk-free asset  $i$  is  $Q^i$ , and therefore,  $Q^f = \sum_{i \in \mathbb{I}} Q^i$ . The investors are equally endowed with these assets at the start of  $t = 0$ .

Since investors are *ex ante* symmetric at  $t = 0$ , we denote the quantity of risk-free asset  $i \in \mathbb{I}$  held by an investor at the end of  $t = 0$  by  $X_1^i$ , while the quantity of the market portfolio is denoted by  $X_1^m$ . The quantity of risk-free asset  $i$  held by an investor type  $s \in \{\underline{S}, \bar{S}\}$  at the end of  $t = 1$  is  $X_{2,s}^i$ , and analogously, for the market portfolio holdings. The prices of the risk-free assets are denoted by  $P_t^i$ , and the price of the risky asset by

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<sup>8</sup>Subtracting  $\sigma^2/2$  from the mean ensures that the expected value of  $\log \varphi^m$  is  $\mu$  and does not depend on  $\sigma^2$ .

$P_t^m$ .

Investors who sell an asset at  $t = 1$  pay a proportional transaction cost  $\tau^i$  for the risk-free asset  $i$  and  $\tau^m$  for the market portfolio.<sup>9</sup> The assumption that transaction costs (interpreted broadly as any form of liquidation cost) are proportional to the market value of assets is an important feature of our model and subsequent analysis. The assumption is empirically relevant in view of bid-ask spreads and the price impact of trading. We assume that for risk-free asset  $i = 1$ ,  $\tau^1 = 0$ . Therefore, there is one risk-free asset without transaction costs in this economy. Otherwise  $\tau^i > 0$  for all  $i \neq 1$ .<sup>10</sup>

Lastly, investors are assumed to have recursive Epstein-Zin (Epstein and Zin, 1989) preferences given by:

$$U_0 = \log(c_0) + E[\chi_s \log(c_{1,s}) + \beta_s \log(U_{2,s})],$$

where  $U_{2,s} = (E[c_{2,s}^{1-\gamma}])^{1/(1-\gamma)}$ , and  $\gamma$  measures the degree of risk aversion. To model the  $t = 1$  liquidity shock of the investors, we assume that

$$\chi_s = \begin{cases} \underline{\chi}, & s = \underline{S} \\ \bar{\chi}, & s = \bar{S} \end{cases}, \text{ with } 1 = \underline{\chi} < \bar{\chi}, \text{ and } \beta_s = \begin{cases} 1, & s = \underline{S} \\ 0, & s = \bar{S}. \end{cases}$$

In what follows, we focus on the case where  $\bar{\chi} = 2$ . This is a convenient parametrization, since, given the number of periods and the assumption of no discounting between periods, it implies that there are no total utility differences between the patient and impatient investors, which gives us tractability at the cost of little loss in generality. We can write the  $t = 0$  preferences of an investor of type  $s$  recursively as

$$U_0 = \log(c_0) + E[U_{1,s}], \tag{1}$$

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<sup>9</sup>In the Appendix we also allow for buyer-specific proportional transaction costs or subsidies.

<sup>10</sup>Since  $\tau$  is a proportional cost, we naturally assume that  $\tau^i \leq 1$  and similar for  $\tau^m$ .

with  $U_{1,s} = \chi_s \log(c_{1,s}) + \beta_s \log(U_{2,s})$  and  $U_{2,s} = \left(E \left[ c_{2,s}^{(1-\gamma)} \right]\right)^{1/(1-\gamma)}$ . At  $t = 1$  we have

$$U_{1,s} = \begin{cases} 2 \log(c_{1,\bar{S}}), & s = \bar{S} \\ \log(c_{1,\underline{S}}) + \log(U_{2,\underline{S}}), & s = \underline{S}. \end{cases} \quad (2)$$

The  $t = 0$  budget constraint of an investor is given by

$$c_0 + \sum_{i \in \mathbb{I}} P_0^i X_1^i + P_0^m X_1^m = Y_0 + \sum_{i \in \mathbb{I}} P_0^i Q^i + P_0^m Q^m. \quad (3)$$

Therefore, the  $t = 1$  budget constraint for a type  $s$  investor is given by

$$\begin{aligned} c_{1,s} + A_{2,s} = Y_1 + A_1 - \sum_{i \in \mathbb{I}} \tau^i P_1^i \max \{0, X_{1,s}^i - X_{2,s}^i\} \\ - \tau^m P_1^m \max \{0, X_{1,s}^m - X_{2,s}^m\}, \end{aligned} \quad (4)$$

where the last two terms on the right-hand side reflect the transaction costs incurred from selling assets, and  $A_1$  denotes the asset holdings the investor has at the beginning of  $t = 1$ , namely  $A_1 \equiv \sum_{i \in \mathbb{I}} P_1^i X_{1,s}^i + P_1^m X_{1,s}^m$ .  $A_{2,s}$  denotes the asset holdings of the investor at the end of  $t = 1$ , with  $A_{2,s} \equiv \sum_{i \in \mathbb{I}} P_1^i X_{2,s}^i + P_1^m X_{2,s}^m$ . Finally, the period 2 budget constraint of a type  $s$  investor in state  $z$  is given by

$$c_{2,s}(z) = \sum_{i \in \mathbb{I}} \delta^i X_{2,s}^i + \varphi^m(z) X_{2,s}^m. \quad (5)$$

Next, we define an equilibrium for this economy.

**Definition 1 (Equilibrium).** *An equilibrium for this economy consists of period 0 asset and consumption choices of the investors,  $\{X_1^i\}_{i \in \mathbb{I}}$ ,  $X_1^m$ , and  $c_0$ ; period 1 asset and period 1 and 2 consumption choices for type  $s$  investors,  $\{X_{2,s}^i\}_{i \in \mathbb{I}}$ ,  $X_{2,s}^m$ ,  $c_{1,s}$ , and  $c_{2,s}(z)$ ; and period 0 and 1 asset prices,  $\{P_0^i\}_i$ ,  $P_0^m$ ,  $\{P_1^i\}_i$ ,  $P_1^m$ , such that*

- a) *given prices, the consumption allocations, and asset holdings solve the period 0 and period 1 problems of the investor (i.e., Eq. (1) subject to the period  $t = 0$  budget constraint in Eq. (3), as well as Eq. (2) subject to the period  $t = 1$  budget constraint in Eq. (4), as well as the period 2 budget constraint in Eq. (5), which must hold for all  $z$ );*
- b) *given the period 0 and period 1 asset holdings of investors, the asset markets clear in both  $t = 0$  and  $t = 1$ , that is*

$$X_1^i = Q^i, \forall i \in \mathbb{I}$$

$$X_1^m = Q^m$$

*and*

$$\lambda X_{2,\bar{S}}^i + (1 - \lambda) X_{2,\underline{S}}^i = Q^i, \forall i \in \mathbb{I}$$

$$\lambda X_{2,\bar{S}}^m + (1 - \lambda) X_{2,\underline{S}}^m = Q^m.$$

## 2.2 Remarks on the main modeling assumptions

Before characterizing the equilibrium, we discuss the two assumptions at the core of our framework – heterogeneous and exogenous asset-specific transaction costs, and an endogenous equilibrium marginal value of liquidity.

**Transaction costs as primitives.** For the most part, in our paper we do not propose a theory of why per-unit transaction costs differ across assets. This is an approach taken by other papers studying asset pricing implications of asset illiquidity, for example Amihud and Mendelson (1986), Vayanos (1998), Vayanos and Vila (1999), Huang (2003), Lo et al. (2004), and Acharya and Pedersen (2005), among others. We take the cost structure  $\{\tau^i\}_{i \in \mathbb{I}}$  and  $\tau^m$  as given and study the equilibrium asset pricing and welfare

implications of its interaction with the demand for liquidity when the marginal value of liquidity is determined in equilibrium. In this respect, our approach differs from models that endogenize per-unit trading costs through asymmetric information frictions (Vayanos and Wang (2012), Kurlat (2013), Malherbe (2014), Bigio (2015)), dealer inventory risk (Ho and Stoll (1981), Grossman and Miller (1988)) or search and matching frictions (Duffie et al. (2005), Coppola et al. (2026)). The advantage of our approach is that we do not have to take a stand on the specific underlying causes of transaction cost heterogeneity which can be quite variable and hard for a single micro-founded theory to explain. Indeed, the empirical evidence indicates that a substantial component of cost heterogeneity across safe assets is idiosyncratic, that is, security-specific and unrelated to differences in cash flows or systematic risk.<sup>11</sup>

**Micro-founding transaction cost heterogeneity.** Transaction costs do, of course, also have a systematic, time-varying component, which rises in volatile periods as dealer intermediation capacity becomes constrained (Duffie (2020), He et al. (2022), Duffie et al. (2023); and see also Hu et al. (2013)). We discuss how this component interacts with our comparative statics in Section 3.5. In addition, in Section 5.1 we develop a theory of transaction cost heterogeneity tailored to our empirical application. There, we decompose the effective cost of liquidating an asset before maturity into a direct market access component and a liquidation risk component, and we show that both components increase with asset maturity.

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<sup>11</sup>The leading example is the on-the-run phenomenon: recently issued Treasuries trade at materially lower costs and higher prices than seasoned issues with nearly identical cash flows (Krishnamurthy (2002), Vayanos and Weill (2008), Pasquariello and Vega (2009), Schneider (2026)). Related phenomena include repo specialness (Duffie (1996), Jordan and Jordan (1997)), benchmark and cheapest-to-deliver status in futures markets, and the persistent valuation differences between Treasury notes and bonds with identical cash flows documented by Musto et al. (2018).

**No period  $t = 2$  transaction costs.** A related modeling choice is that no trading takes place after  $t = 1$ , so anticipated future trading costs play no role. Allowing for an additional round of trade before payoffs are realized would reintroduce the standard channel whereby expected future transaction costs are capitalized into current prices (Amihud and Mendelson, 1986). In our framework this is equivalent to the heterogeneity in the period  $t = 2$  payoffs  $\delta^i$ , generating price differences in earlier periods.

**Supply- and payoff-dependent transaction costs.** Our framework extends naturally to transaction costs that depend on other elements of the asset structure itself. For example, the per-unit cost can be allowed to depend on the asset's supply,  $\tau^i = \tau^i(Q^i)$ , capturing potentially non-linear effects of supply on market depth (Coppola et al., 2026): a larger float can thicken the market and lower trading costs, whereas a supply that is large relative to dealers' intermediation capacity strains market depth and raises them (D'Amico and King (2013), Lou et al. (2013), Duffie (2020), He et al. (2022), Duffie et al. (2023)). The cost could also depend on the asset's terminal payoff,  $\tau^i = \tau^i(\delta^i)$ , via a *buyer attractiveness channel*: an asset with a higher per-asset payoff may draw a broader pool of potential buyers, also thickening markets. Our equilibrium characterization would go through with  $\tau^i(Q^i, \delta^i)$  in place of  $\tau^i$ , while comparative statics with respect to supplies or payoffs acquire an additional term, reflecting the induced movement in the per-unit cost, and our qualitative conclusions continue to hold whenever the elasticity of  $\tau^i$  with respect to its arguments is sufficiently small.<sup>12</sup>

**Endogenous marginal value of liquidity.** The presence of curvature in utility from period  $t = 1$  consumption, or equivalently a finite elasticity of intertemporal substitution in our specification, makes the marginal value of liquidity endogenous to transaction

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<sup>12</sup>For example, with  $\tau^i(Q^i) = \tau_0^i + \kappa^i g(Q^i)$  for an increasing function  $g$ , the effect of a change in  $Q^i$  on the average transaction cost decomposes into the composition effect of Proposition 4 and a market depth effect proportional to  $\kappa^i g'(Q^i) Q^i$ ; the former dominates for  $\kappa^i$  sufficiently small.

cost expenditures. If period utility were linear at  $t = 1$ , this marginal value would instead be fixed and transaction costs would affect relative returns only through the direct asset-specific trading-cost channel, as in the standard Amihud and Mendelson (1986) mechanism. In that benchmark, there are no interactions across assets through the marginal value of liquidity: a change in the supply or transaction cost of one asset does not affect the relative returns on other assets through this channel.

## 2.3 Characterization

We solve the model backwards in time. We first look at the  $t = 1$  problems and market clearing conditions given the asset holdings of the investors at the beginning of  $t = 1$  and then move to the  $t = 0$  problem and market clearing conditions given some anticipated  $t = 1$  prices. Imposing rational expectations and given the  $t = 0$  choices of agents, this approach characterizes the equilibrium of this economy.

### 2.3.1 $t = 1$ characterization

From  $t = 1$  onwards, there are two types of investors: patient and impatient. An impatient investor sells all her assets because she derives no utility from consumption in future periods. A patient investor's problem can be split into a consumption-saving problem and a portfolio choice problem.<sup>13</sup> In the consumption-saving problem, she decides how much to consume in the current period and how much to save for the next period. In the portfolio choice problem, she decides how to allocate her savings over assets. Specifically, below we denote by  $w$  the portfolio share invested in the market portfolio. Market clearing implies that patient investors hold all assets at the end of  $t = 1$ . The following Lemma summarizes the  $t = 1$  characterization.

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<sup>13</sup>This is implied by the assertion that in equilibrium she only increases her asset holdings at  $t = 1$ , as we verify in the Appendix.

**Lemma 1** (*t = 1 characterization*). Let  $A_1 = \bar{P}_1^f \bar{Q}_1^f + P_1^m Q_1^m$  denote the value of an investor's financial asset holdings at  $t = 1$  with  $\bar{P}_1^f = \frac{P_1^i}{\delta^i}$ ,  $\bar{Q}_1^i = \delta^i Q_1^i$  and  $\bar{Q}_1^f = \sum_{i \in \mathbb{I}} \bar{Q}_1^i$ ,  $\forall i \in \mathbb{I}$ . The following characterizes the equilibrium prices and allocations at  $t = 1$  and  $t = 2$ .

a) Investors'  $t = 1$  consumption and saving decisions:

$$c_{1,\bar{S}} = \frac{2}{1+\lambda} Y_1 - \sum_{i \in \mathbb{I}} \tau^i P_1^i Q^i - \tau^m P_1^m Q^m$$

$$c_{1,\underline{S}} = \frac{1}{1+\lambda} Y_1$$

and

$$A_{2,\bar{S}} = 0$$

$$A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}.$$

b) Patient investors'  $t = 2$  portfolio allocations solve  $\sum_{i \in \mathbb{I}} P_1^i X_{2,\underline{S}}^i = (1-w) A_{2,\underline{S}}$  and  $P_1^m X_{2,\underline{S}}^m = w A_{2,\underline{S}}$ , where  $w$  solves:

$$V_2(A_{2,\underline{S}}) = \max_w \left( E \left[ c_{2,\underline{S}}(z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)}$$

*s.t.*

$$c_{2,\underline{S}}(z) = \left[ (1-w) \frac{1}{\bar{P}_1^f} + w \frac{\varphi^m(z)}{P_1^m} \right] A_{2,\underline{S}}, \quad \forall z.$$

c) Prices are determined by:

$$\bar{P}_1^f \bar{Q}^f = (1-w) \frac{1-\lambda}{1+\lambda} Y_1$$

$$P_1^m Q^m = w \frac{1-\lambda}{1+\lambda} Y_1,$$

where  $w$ , with  $0 \leq w \leq 1$ , is the equilibrium portfolio share invested by patient investors in the market portfolio.

*Proof.* See Appendix A.2. □

### 2.3.2 $t = 0$ characterization

At  $t = 0$ , investors choose their current consumption and their asset portfolio. The problem is characterized by a standard Euler equation. The marginal utility of today's consumption must equal the expected marginal utility of tomorrow's consumption times the return on each asset earned by holding it from the first to the second period, where the expectation is taken over the idiosyncratic investor-specific state at  $t = 1$ . All uninvested resources are consumed, and there is asset market clearing, with all assets held by the investors. Lemma 2 summarizes the equilibrium equations that characterize the economy at  $t = 0$ .

**Lemma 2** ( $t = 0$  characterization). *Given anticipated period  $t = 1$  asset prices  $P_1^i, \forall i \in \mathbb{I}$  and  $P_1^m$ , the following list of equations characterizes the economy at  $t = 0$ :*

a) *Prices are determined by the Euler equations with respect to every asset:*

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}} \frac{P_1^j}{P_0^j} (1 - \tau^j) + (1 - \lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^j}{P_0^j}, \quad \forall j \in \mathbb{I} \cup m.$$

b) *The portfolio allocations, or asset holdings, are:*

$$X_1^i = Q^i, \quad \forall i \in \mathbb{I}$$

$$X_1^m = Q^m.$$

c) *Consumption is given by:*

$$c_0 = Y_0.$$

*Proof.* See Appendix A.2. □

## 2.4 Approximating the period $t = 1$ portfolio choice problem

For some of our theoretical results below, we will simplify the  $t = 1$  portfolio choice problem of the patient investor by applying a log-normal approximation for the equilibrium asset returns, as in Iachan et al. (2021), which in turn is based on the log-normal portfolio choice approximation in Campbell and Viceira (2002). Under that approximation, the portfolio choice problem of the patient investor simplifies to a mean-variance portfolio choice problem, given normally distributed log asset returns.

To set up the simplified problem, we define the log returns  $r_2^m(z) \equiv \log R_2^m = \log\left(\frac{\varphi^m(z)}{\bar{P}_1^m}\right)$  and  $r_2^f \equiv \log R_2^f = \log\left(\frac{1}{\bar{P}_1^f}\right)$ . Also, we define the log of the certainty-equivalent return on the portfolio,  $R_2^{ce}$ , by  $r_2^{ce} \equiv \log(R_2^{ce})$ , and the log of the risk premium on the risky asset by

$$\pi \equiv \log E[R_2^m] - \log R_2^f = \log E[\varphi^m(z)] + \log \frac{\bar{P}_1^f}{\bar{P}_1^m}.$$

The approximate solution of the portfolio choice problem is then given by

$$r_2^{ce} - r_2^f = \max_w w\pi - \frac{\gamma}{2}w^2\sigma^2.$$

The first-order condition gives the Merton share (Merton, 1969):

$$w = \frac{\pi}{\gamma\sigma^2}. \tag{6}$$

The share invested in the risky asset is proportional to the Sharpe ratio,  $\pi/\sigma$ , where the coefficient of proportionality depends inversely on the risk aversion coefficient  $\gamma$ . The certainty equivalent log return of the investor's portfolio is then

$$r_2^{ce} = r_2^f + \frac{1}{2} \frac{\pi^2}{\gamma \sigma^2}. \quad (7)$$

This approximation requires a minor adjustment in our equilibrium definition as well. In particular, the investor is assumed to act according to Eqs. (6) and (7) when forming  $t = 1$  portfolios and making her  $t = 0$  and  $t = 1$  consumption-saving decisions.

Finally, note that

$$\pi \approx \log E[\varphi^m(z)] + \frac{\bar{P}_1^f - P_1^m}{P_1^m} \quad (8)$$

whenever  $\frac{\bar{P}_1^f - P_1^m}{P_1^m}$  is close to zero, so that  $\log \frac{\bar{P}_1^f}{P_1^m} \approx \frac{\bar{P}_1^f - P_1^m}{P_1^m}$ . This will be the case whenever the expected payoff of the market portfolio  $E[\varphi^m(z)]$  is close to the terminal payoff on the risk-free assets and the risk premium is relatively small. We regard this as empirically plausible, given the observed risk premia on risky assets in the macroeconomy, and will therefore use this approximation for the log risk premium whenever we derive results involving a positive supply of risky assets below.

We point out that from our main theoretical results below only Propositions 6 and 7 and the second case in Proposition 5 depend on this log-approximation. The other results are independent of it, and are derived exactly.

## 3 Theoretical results

### 3.1 The value of convenience

An important equilibrium object that is central to our theoretical results is *the value of convenience*. To define this object, let  $m_{1,s} \equiv \frac{\partial U_{1,s}}{\partial c_{1,s}} / \frac{\partial U_0}{\partial c_0}$  denote the ratio of marginal

utilities of consumption for an investor of type  $s \in \{\underline{S}, \bar{S}\}$ . Define

$$\eta \equiv \frac{m_{1,\bar{S}}}{m_{1,\underline{S}}} = \frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}} / \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}. \quad (9)$$

Therefore,  $\eta$  captures the degree to which period  $t = 1$  marginal utilities of consumption between patient and impatient investors are aligned in equilibrium. Indeed, as we show in Section 4, in the first-best full liquidity risk-sharing case  $\eta = 1$ , and there is full equalization of marginal utilities across idiosyncratic states at  $t = 1$ . Moreover, the higher  $\eta$  is, the further away this economy is from the full-insurance benchmark. In our decentralized economy, generally  $\eta \geq 1$ , with the value of  $\eta$  determined in equilibrium. Therefore,  $\eta$  can be thought of as reflecting the equilibrium liquidity scarcity in this economy. For this reason we refer to  $\eta$  as the *value of convenience*.

**Proposition 1 (Value of Convenience).** *The value of convenience  $\eta$  is given by*

$$\eta = \frac{\frac{2}{1+\lambda} Y_1}{\frac{2}{1+\lambda} Y_1 - \Omega} \quad (10)$$

with  $\Omega \equiv \sum_{i \in \mathbb{I}} \tau^i P_1^i Q^i + \tau^m P_1^m Q^m$ .

*Proof.* We use the investors'  $t = 1$  consumption specified in Lemma 1 and insert it in the derivative of the logarithmic utility function given by  $\frac{1}{c_{1,s}}$  for  $s = \bar{S}, \underline{S}$ . Taking the ratio according to Eq. (9) yields Eq. (10).  $\square$

The value of convenience is a function of model primitives and asset prices.  $\Omega$  gives the total transaction cost expenditure that an impatient seller incurs in equilibrium. We can rewrite it to  $\Omega = \bar{\tau} \bar{P}_1^f \bar{Q}^f + \tau^m P_1^m Q^m$  with  $\bar{\tau} \equiv \sum_i \frac{\bar{Q}^i}{\bar{Q}^f} \tau^i$ .  $\bar{\tau}$  is the (weighted) average transaction cost on risk-free assets in this economy. The weights are given by the payoff-adjusted volume weights. The transaction cost expenditure creates a wedge in the marginal utilities of patient and impatient investors. The larger the expenditure, the

larger the wedge and the higher the value of convenience  $\eta$ . Since the transaction cost expenditure depends on asset prices, which are endogenously determined in equilibrium, changes in these prices would impact the wedge in marginal utilities and the degree of risk-sharing in equilibrium. It is precisely this dependence of the cost bill on endogenous prices that price-taking investors fail to internalize, generating the pecuniary externality we analyze in Section 4.

We can further simplify the expression for  $\eta$  by using the equilibrium prices from Lemma 1. Specifically, the equilibrium transaction cost expenditure is  $\Omega = [\bar{\tau}(1 - w) + \tau^m w](1 - \lambda)Y_1/(1 + \lambda)$ . Substituting into Eq. (10) and simplifying, we get

$$\eta = \frac{1}{1 - (1 - \lambda)[\bar{\tau}(1 - w) + \tau^m w]/2}. \quad (11)$$

### 3.2 Relative inconvenience yields

We compare the return on a specific asset *relative* to the return on the *most liquid* risk-free asset in the economy. A higher relative return means that the most liquid asset commands a premium (a higher convenience yield) over that specific asset.<sup>14</sup> Below, for brevity, we will use the term *relative inconvenience yield* when referring to the (positive) relative return of a specific asset and the most liquid risk-free asset.

We define the period  $t = 1$  gross return on an asset  $i \in \mathbb{I} \cup \{m\}$  as  $R_1^i \equiv \frac{P_1^i}{P_0^i}$  and show the following result:

**Proposition 2 (Relative inconvenience yields).** *Define  $\psi^j \equiv \frac{R_1^j}{R_1^1}$  as the return on asset  $j$  relative to the return on the most liquid asset in the economy, risk-free asset 1. Then*

$$\psi^i = \frac{1}{1 - \lambda\tau^i - m_{1,\bar{S}}\tau^i R_1^1} = \frac{1}{1 - \tau^i \frac{\lambda}{\lambda + (1 - \lambda)/\eta}} \text{ for all } i \in \mathbb{I} \cup m.$$

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<sup>14</sup>Note that we are consistent with standard measures of convenience yields, which are always expressed in terms of *relative* returns or spreads.

*Proof.* See Appendix A.2. □

Proposition 2 shows that the relative inconvenience yield  $\psi^i$  is influenced by three forces. First, an asset's return has to compensate for the transaction cost incurred by selling the asset. Second, an asset's return has to compensate for the transaction cost being incurred *precisely when* the investor is impatient. Third, how much the transaction cost matters for the relative inconvenience yield depends on the value of convenience,  $\eta$ . A lower value of convenience also lowers the relative inconvenience yields on all assets.

### 3.3 Effects of transaction costs

We next show how a change in an asset's transaction cost impacts the relative inconvenience yields across the entire cross-section of assets in the economy.

**Proposition 3 (Transaction costs).** *The relative inconvenience yield of asset  $i$ ,  $\psi^i$ , is increasing in that asset's transaction cost,  $\tau^i$ , i.e.,  $\frac{\partial \psi^i}{\partial \tau^i} > 0$ ,  $\forall i \in (\mathbb{I} \setminus \{1\}) \cup m$ . It is also increasing in the transaction cost of any other asset,  $\tau^k$ , i.e.,  $\frac{\partial \psi^i}{\partial \tau^k} > 0$ , for  $k \neq i$  and  $i \in (\mathbb{I} \setminus \{1\}) \cup m$ . The higher is  $\tau^i$ , the higher is the impact, i.e.,  $\frac{\partial^2 \psi^i}{\partial \tau^{i^2}} > 0$  and  $\frac{\partial^2 \psi^i}{\partial \tau^k \partial \tau^i} > 0$ ,  $\forall i \in (\mathbb{I} \setminus \{1\}) \cup m$ .*

*Proof.* See Appendix A.2. □

Therefore, a change in an asset's transaction cost not only impacts its own relative inconvenience yield but also the entire cross-section of asset returns. This spillover effect is driven by a change in the value of convenience  $\eta$ . An increase in the transaction cost of any asset *increases* the value of convenience, since the transaction cost expenditure of sellers increases, decreasing risk-sharing opportunities. A higher value of convenience affects all assets and raises all relative inconvenience yields, making the most liquid asset relatively more valuable. The more illiquid an asset is in terms of its transaction cost, the more its corresponding relative inconvenience yield is affected.

### 3.4 Effects of changes in asset supplies

We next show how changes to the composition and overall supply of safe assets affect the cross-section of asset returns. We first consider a compositional change in the supply of risk-free assets due to an increase in the supply of one asset and an equivalent decrease in the supply of another asset.

**Proposition 4 (Purifying/polluting effects).** *Given a compositional change where  $d\bar{Q}^l = -d\bar{Q}^k > 0$  for two risk-free assets  $k$  and  $l$ , the relative inconvenience yield  $\psi^i$  for any asset  $i \in (\mathbb{I} \setminus \{1\}) \cup m$  decreases strictly in response to this change, iff  $\tau^l < \tau^k$ .*

*Proof.* See Appendix A.2. □

Substituting more liquid for less liquid assets one-for-one can be interpreted as a “purification” of the pool of safe assets by altering its composition. This decreases the value of convenience and compresses relative inconvenience yields. We call this the “purifying/polluting effect” of safe asset supply.

Next we show the asset pricing implications of changes in the overall supply of risk-free assets, keeping the composition of risk-free assets fixed. In Proposition 5 we first assume that there is no risky asset in the economy,  $Q^m = 0$ . In a second step we use the log-approximation from Section 2.4 to also discuss the case with  $Q^m > 0$ . In that case the supply of risk-free assets also impacts the equilibrium risk premium.

**Proposition 5 (Total safe asset supply effect).** *Consider a change in  $\bar{Q}^f$ , keeping the composition of risk-free assets fixed:*

- *If  $Q^m = 0$ , then  $\frac{\partial \psi^i}{\partial \bar{Q}^f} = 0$ , for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$ .*
- *If  $Q^m > 0$  and given the log-approximation of the investor’s portfolio problem and the risk premium, then  $\frac{\partial \psi^i}{\partial \bar{Q}^f} \leq 0$ , for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$ , iff  $(\tau^m - \bar{\tau}) \geq 0$ .*

*Proof.* See Appendix A.2. □

If there are no risky assets, an increase in the aggregate supply of safe assets, holding their composition fixed, does not affect inconvenience yields. This is a direct consequence of having log intertemporal preferences and an elasticity of intertemporal substitution (EIS) of unity. In that case, income and substitution effects from intertemporal price changes cancel out and the investor saves a constant share of her wealth. In equilibrium, that leads to an equilibrium value of the risk-free assets that is independent of asset supply. Consequently, the value of convenience remains unchanged, and so do relative inconvenience yields.

This conclusion changes once risky assets are present. Let  $\tau^m > \bar{\tau}$ , so risk-free assets are on average more liquid than the risky asset, which we view as the empirically relevant case. A higher supply of risk-free assets decreases the equilibrium investors' portfolio weight  $w$  and a higher share is invested into risk-free assets. The transaction cost expenditure of impatient investors decreases and, by Eq. (11), so does the value of convenience. Consequently, the cross-section of relative inconvenience yields also decreases.

Propositions 4 and 5 can be used to characterize the effect of a change in the supply of *a single* risk-free asset on relative inconvenience yields. That effect is the combination of an increase in the overall supply of risk-free assets and a number of compositional changes, since an increase in the supply of a single risk-free asset increases its relative weight in the pool of risk-free assets and decreases the relative weight of all other assets. For example, suppose that  $Q^m = 0$  and there are two risk-free assets. In the simple case without risky assets, only the compositional channel operates: inconvenience yields rise if the injected asset has higher transaction costs compared to the average (a polluting effect); otherwise, they fall. In the general case with risky assets both channels are active. Inconvenience yields decrease if the injected asset has transaction costs below average. Otherwise it is unclear which channel dominates.

### 3.5 Effects of payoff risk

Our model makes predictions of how changes in aggregate payoff risk during crisis and “flight to safety” episodes impact the cross-section of safe asset returns. The following result characterizes these effects through the lens of our model:

**Proposition 6 (“Flight to safety” revaluation effects).** *Given the log-approximation of the investor’s portfolio problem and the log risk premium, then, for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$ ,  $\frac{\partial \psi^i}{\partial \sigma^2} < 0$ , iff  $\tau^m > \bar{\tau}$ , and  $\frac{\partial \psi^i}{\partial \gamma} < 0$ , iff  $\tau^m > \bar{\tau}$ .*

*Proof.* See Appendix A.2. □

An increase in the payoff risk  $\sigma^2$  or the risk aversion parameter  $\gamma$  raises the price of the risk-free assets and lowers the price of the risky asset. This tilts the equilibrium portfolio weight away from the risky asset and toward the risk-free assets. If safe assets are on average more liquid than the risky asset, which we view as the empirically relevant case, this revaluation lowers the total transaction cost expenditure of impatient sellers. By Equation (11), the value of convenience then decreases, and so do the relative inconvenience yields across the whole cross-section of safe assets.

This prediction may at first appear to contradict the conventional view that periods of elevated risk are usually seen as periods of high convenience yields. Two clarifications sharpen this seemingly counter-factual prediction. First, common empirical proxies for convenience yields are spreads against *risky* or defaultable benchmarks – for instance, the AAA-Treasury spread of Krishnamurthy and Vissing-Jorgensen (2012). A payoff-risk shock increases such spreads mechanically through the revaluation of, and the higher required compensation for, the risky asset, in addition to any change in liquidity premia.

Another reason for the divergence with the conventional view is that in addition to an increase in payoff risk, there is likely a second force that operates in crises that our Proposition holds fixed: the incidence of liquidity shocks itself. During periods of market

stress, margin calls, redemptions, and funding pressures plausibly raise transaction costs  $\tau^i$  or the share of investors who are forced to liquidate, so that  $\lambda$  increases with aggregate risk.<sup>15</sup> The following result incorporates this channel into our framework and characterizes the resulting offsetting effect on relative inconvenience yields.

**Proposition 7 (Crisis liquidity demand).** *Given the log-approximation of the investor's portfolio problem and the log risk premium, suppose that the share of impatient investors increases with payoff risk,  $\lambda = \lambda(\sigma^2)$  with  $\lambda'(\sigma^2) \geq 0$ . Then, for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$ :*

*i)  $\frac{d\psi^i}{d\sigma^2} = \frac{\partial\psi^i}{\partial\sigma^2} + \frac{\partial\psi^i}{\partial\lambda}\lambda'(\sigma^2)$ , where the first (revaluation) term is negative iff  $\tau^m > \bar{\tau}$ , as in Proposition 6, and the second (liquidity-demand) term is positive;*

*ii) there exists a threshold  $\bar{\xi} > 0$ , common to all assets, such that  $\frac{d\psi^i}{d\sigma^2} > 0$  for every  $i \in (\mathbb{I} \setminus \{1\}) \cup m$  iff  $\lambda'(\sigma^2) > \bar{\xi}$ .*

*An analogous result holds when  $\lambda$  is an increasing function of  $\gamma$ .*

*Proof.* See Appendix A.2. □

Therefore, a sufficiently strong liquidity-demand response reverses the sign of the comparative static from Proposition 6 for the *entire* cross-section of relative inconvenience yields at once. Whether crises compress or widen the cross-section of safe asset returns is therefore an empirical question about the strength of the liquidity demand relative to the revaluation effect. In our empirical application in Section 5 we aim to hold this liquidity-demand channel fixed by controlling for the MOVE Index, so that the coefficient on the VIX Index isolates the revaluation effect of changes in  $\sigma^2$  and  $\gamma$ , as in Proposition 6.

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<sup>15</sup>Brunnermeier et al. (2024) emphasize precisely this countercyclicality of uninsurable idiosyncratic risk, which in their framework gives safe assets a negative  $\beta$ .

## 4 Welfare and optimal policy

In this section we show that the value of convenience is a measure of welfare in this economy, since it summarizes the degree of liquidity risk-sharing among agents that is achieved in equilibrium. If the value of convenience is greater than one, the decentralized equilibrium leads to a suboptimal level of liquidity risk-sharing. The reason for this suboptimality is a pecuniary externality, since asset prices enter the transaction cost expenditure of asset sellers, which agents fail to internalize when forming their portfolios. In light of this result, we discuss welfare-improving policies.

### 4.1 The value of convenience as a welfare measure

As already alluded to in Section 3.1, the value of convenience is a welfare measure for this economy, with higher values of  $\eta$  implying lower aggregate welfare. To show this formally, we characterize the welfare maximization problem of a utilitarian social planner that maximizes the *ex ante* ( $t = 0$ ) expected utility of an investor.<sup>16</sup> Specifically, the planner maximizes aggregate welfare,  $W = E[U_0]$ , by solving

$$\begin{aligned} \max_{c_{1,\bar{S}}, c_{1,\underline{S}}} \quad & \log Y_0 + \lambda 2 \log c_{1,\bar{S}} \\ & + (1 - \lambda) \log c_{1,\underline{S}} + (1 - \lambda) \log \left( E \left[ \left( \sum_i \delta^i Q^i + \varphi_2^m(z) Q^m \right)^{1-\gamma} \right] \right)^{\frac{1}{(1-\gamma)}} \\ \text{s.t.} \quad & \lambda c_{1,\bar{S}} + (1 - \lambda) c_{1,\underline{S}} = Y_1, \end{aligned}$$

where we have substituted for the  $t = 0$  and  $t = 2$  endowments. The optimal period  $t = 1$  consumption allocation equalizes the marginal utilities of the patient and impatient investors and is thus given by  $c_{1,\bar{S}} = 2 \frac{Y_1}{(1+\lambda)}$  and  $c_{1,\underline{S}} = \frac{Y_1}{(1+\lambda)}$ . Therefore, from Eq. (9), we have that  $1 = \frac{1}{c_{1,\underline{S}}} / \frac{2}{c_{1,\bar{S}}} = \eta$ . Moreover, we can express welfare  $W$  as a function of  $\eta$ ,

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<sup>16</sup>Since the planner cannot shift resources across periods it must be that  $c_0 = Y_0$ .

which is uniquely maximized at  $\eta = 1$  (full liquidity risk-sharing), with  $W$  monotonically decreasing in  $\eta$  for  $\eta \geq 1$ .

**Proposition 8 (Welfare).** *For  $\eta \geq 1$ , welfare  $W$  is monotone decreasing in the value of convenience,  $\eta$ ,  $\frac{\partial W}{\partial \eta} < 0$ , and  $W$  is maximized for  $\eta = 1$  (full liquidity risk-sharing).*

*Proof.* See Appendix A.2. □

Given the decentralized equilibrium value of  $\eta$  in this economy (cf. Eq (10)), it follows directly that with positive transaction costs  $\eta > 1$  and the competitive equilibrium does not achieve the first-best level of risk-sharing and welfare. The reason for this inefficiency is a pecuniary externality, which we discuss next.

## 4.2 Pecuniary externality

Consider a simplified version of our model economy with no payoff risk (no risky asset) and only two risk-free assets with unit payoffs ( $\delta^1 = \delta^2 = 1$ ). The first asset has zero transaction costs, while the second asset has a transaction cost given by  $\tau$ . Moreover, suppose that both assets have an equal supply of  $Q^f/2$ .

Consider the period  $t = 1$  portfolio choice of a patient investor. The asset holdings,  $X_{2,\underline{S}}^i$ , for any of the safe assets satisfy the Euler equations

$$\frac{1}{c_{2,\underline{S}}} = \frac{P_1^i}{c_{1,\underline{S}}}, \quad i \in \{1, 2\}, \quad (12)$$

together with the period  $t = 1$  and  $t = 2$  budget constraints

$$\begin{aligned} c_{1,\underline{S}} + \sum_{i \in \{1,2\}} P_1^i X_{2,\underline{S}}^i &= Y_1 + \sum_{i \in \{1,2\}} P_1^i X_1^i, \\ c_{2,\underline{S}} &= \sum_{i \in \{1,2\}} X_{2,\underline{S}}^i. \end{aligned}$$

The Euler equations and market clearing imply that, in equilibrium, period  $t = 1$  asset holdings  $X_{2,\underline{S}}^i$  and asset prices  $P_1^i$  are such that  $P_1^1 = P_1^2 \equiv P_1^f$ . Market clearing, in addition, implies that  $X_{2,\underline{S}}^i = Q^f/2(1 - \lambda)$ , which from the period budget constraints pins down period 1 and 2 equilibrium consumption and the common price  $P_1^f = c_{1,\underline{S}}/c_{2,\underline{S}} = (1 - \lambda)Y_1/[(1 + \lambda)Q^f]$ . Finally, from Proposition 1, the value of convenience  $\eta$  is  $\eta = \frac{2}{1+\lambda}Y_1/(\frac{2}{1+\lambda}Y_1 - \Omega)$ , where the transaction cost expenditure is  $\Omega = \tau P_1^f Q^f/2$ .

Suppose that we distort the asset demand schedules of the patient investors away from the privately optimal levels given by the above Euler equations and the period budget constraints (possibly also distorting period 1 consumption). Specifically, suppose that we distort  $X_{2,\underline{S}}^1$  up and  $X_{2,\underline{S}}^2$  down, as if asset 1 has a period 2 payoff of  $1 + \rho > 1$  and asset 2 has a period 2 payoff of  $1 - \rho < 1$  for some  $\rho > 0$ . Put differently, the distorted asset demand schedules (and possibly distorted period 1 consumption) are such that they are at an interior value if the following distorted Euler equations hold:

$$\begin{aligned} \frac{1 + \rho}{c_{2,\underline{S}}} &= \frac{P_1^1}{c_{1,\underline{S}}}, \\ \frac{1 - \rho}{c_{2,\underline{S}}} &= \frac{P_1^2}{c_{1,\underline{S}}}. \end{aligned} \tag{13}$$

Essentially, for small values of  $\rho$ , such that the response of period 1 consumption is second order, what this demand distortion does is to shift out the demand curve for asset 1 and shift in the demand curve for asset 2.

How do these distorted demand schedules impact equilibrium asset prices? It will no longer be the case that  $P_1^1 = P_1^2$ . Since market clearing implies that equilibrium asset prices must be such that the investors hold both assets in equilibrium, it follows that  $P_1^1$  and  $P_1^2$  must satisfy the modified Euler equations

$$\frac{1}{c_{2,\underline{S}}} = \frac{\overline{P}_1^i}{c_{1,\underline{S}}}, \quad i \in \{1, 2\}, \tag{14}$$

where  $\bar{P}_1^1 = P_1^1 / (1 + \rho)$  and  $\bar{P}_1^2 = P_1^2 / (1 - \rho)$ . Therefore,  $\bar{P}_1^1 = \bar{P}_1^2 = \bar{P}_1^f$ , where from market clearing and the modified Euler equation,  $\bar{P}_1^f = (1 - \lambda) Y_1 / [(1 + \lambda) Q^f]$ . Note that  $\bar{P}_1^f$  equals  $P_1^f$  in the undistorted economy. Intuitively, the symmetric distortion and equal asset supply imply that only the relative prices of the two assets change, while the aggregate value of the assets stays the same.

These equilibrium asset prices give a value of convenience of  $\tilde{\eta} = \frac{2}{1+\lambda} Y_1 / \left( \frac{2}{1+\lambda} Y_1 - \tilde{\Omega} \right)$ , where the transaction cost expenditure is  $\tilde{\Omega} = P_1^2 \tau Q^2 = (1 - \rho) P_1^f \tau Q^f / 2 < \Omega$ . Therefore,  $\tilde{\eta} < \eta$  and so welfare in the economy with distorted asset demand is higher than welfare in the undistorted economy.

The reason why the distortion in investors' asset demand away from their privately optimal demand improves welfare is that the competitive equilibrium level of liquidity risk-sharing is suboptimal. There is a *pecuniary externality* caused by asset prices entering the transaction cost bill. Patient agents, who purchase the assets in period  $t = 1$  and act as price takers, fail to internalize the impact of their portfolio choices on the transaction costs the selling impatient agents pay. Since patient agents are not directly affected by the transaction costs, they treat all assets as equally valuable. But higher transaction cost expenditures due to higher asset values of the more illiquid assets increase the value of convenience  $\eta$  and decrease welfare. From a social optimum perspective, the more liquid asset is *more valuable* because it carries a lower transaction cost. Therefore, the distortion in asset demand brings the private and social value of the two assets closer together. As a result, the transaction cost bill decreases and aggregate welfare in the economy increases. In light of this pecuniary externality, we next discuss possible welfare-improving policies.

## 4.3 Welfare-improving policies

### 4.3.1 Asset payoff management

Assume for simplicity there is no risky asset. Building on the two-asset example above, we show that by increasing the payoff on the most liquid risk-free asset the policy maker can decrease the equilibrium value of convenience,  $\eta$ , and increase welfare. We assume that the higher payoff, call it  $\tilde{\delta}^1$  is financed with a tax on the period  $t = 2$  payoffs of all other safe assets, which reduces their effective payoff to  $\tilde{\delta}^i < \delta^i$ . Lastly, we assume that revenue neutrality holds so that  $\sum_i \tilde{\delta}^i Q^i = \bar{Q}^f$ .

**Proposition 9 (Payoff management).** *Consider the revenue-neutral set of payoff taxes and subsidies described above. Then  $\left. \frac{\partial \eta}{\partial \tilde{\delta}^1} \right|_{\sum_i \tilde{\delta}^i Q^i = \bar{Q}^f} < 0$ .*

*Proof.* See Appendix A.2. □

To provide additional intuition for this result, note first that in the model the period  $t = 1$  equilibrium price of asset  $i$  is given by

$$P_1^i = \delta^i \frac{1 - \lambda Y_1}{1 + \lambda \bar{Q}^f}. \quad (15)$$

This equation arises from the fact that in equilibrium all safe assets have an equal payoff-adjusted  $t = 1$  price. Consumption smoothing by the marginal buyer combined with market clearing pins down that payoff-adjusted common price. A higher period  $t = 2$  payoff,  $\delta^i$ , increases the relative price of asset  $i$  and hence the equilibrium value of that asset. If we increase the payoff of the most liquid risk-free asset as described above, then this valuation effect lowers the transaction cost expenditure of impatient sellers. This is similar to the compositional changes from Proposition 4 and the valuation effect from Proposition 6, where higher payoff risk decreases the equilibrium value of the relatively illiquid risky asset and increases the equilibrium value of the relatively more liquid risk-

free assets. Therefore, an increase in the effective payoffs of more liquid safe assets, for example, via preferential tax treatment of such assets, can improve liquidity risk-sharing and decrease inconvenience yields in the economy.

### 4.3.2 Asset supply management

An alternative set of policies involves managing the quantities and composition of the different risk-free assets. Following Proposition 4, a policy that replaces relatively less liquid with relatively more liquid assets would reduce the equilibrium value of convenience  $\eta$ . Such a policy can be broadly interpreted as conventional central bank liquidity provision or an unconventional balance sheet policy, such as Operation Twist (Greenwood and Vayanos, 2010) or quantitative easing (QE) programs (Krishnamurthy and Vissing-Jorgensen, 2011). An alternative interpretation for such a policy is the Treasury managing the maturity structure of government debt by issuing relatively more shorter maturity government debt. Indeed, as shown by Bi et al. (2026), Treasury maturity extension shocks that keep the total supply of Treasury debt fixed tend to reduce the convenience yields at the long end.

Since both policies involve substituting more liquid assets (i.e., central bank reserves or Treasury bills) for less liquid assets, according to our simple theory such interventions would decrease relative inconvenience yields and the value of convenience in the economy. The US Treasury Buyback Program provides a recent, concrete example of a government debt policy designed to withdraw illiquid assets from the market. By conducting buybacks, the Treasury allows primary dealers to return less liquid off-the-run Treasuries. This alleviates balance sheet strains and fosters liquidity in the secondary market (Zhou, 2025).

Nevertheless, these asset supply management results only pertain to liquidity effects and may easily be overturned in a richer model – for example, if there were other reasons

why investors demand safety in addition to liquidity demand, such as demand for longer duration assets (e.g., Vayanos and Vila (2021)) or regulatory reasons (Corell et al., 2024). In that case, a central bank that buys longer maturity Treasury notes and bonds and replaces them with very short maturity, highly liquid central bank reserves or a Treasury that issues more Treasury bills and less Treasury notes and bonds may actually end up worsening welfare for some investors in the economy by exacerbating the scarcity of long-duration assets.<sup>17</sup>

## 5 Empirical evidence

In this section we use data for the US Treasury market to document a new set of stylized facts, which we interpret through the lens of our theoretical framework. Since our empirical investigation examines relative convenience yields for assets of different maturity, we first develop a modified version of our framework that micro-founds variation in transaction costs as arising from differences in asset maturity. After that we proceed to explain our measure of relative convenience yields before showing the new stylized facts.

### 5.1 Transaction costs and asset maturity

Why can differences in maturity lead to differences in effective liquidation costs? Following Acharya and Pedersen (2005), one can think of the liquidation cost as comprising of two components,

$$\tau^i = \tau_{\text{dir}}^i + \tau_{\text{risk}}^i, \quad (16)$$

where  $\tau_{\text{dir}}^i$  is the direct market access cost of transacting in the secondary market and potentially unwinding any associated derivative position, and  $\tau_{\text{risk}}^i$  is the liquidation risk

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<sup>17</sup>In addition, the discussion above abstracts completely from the costs associated with the central bank assuming payoff risk or the Treasury getting more exposed to rollover risk.

premium arising from the covariance between an asset’s transaction cost and the aggregate degree of liquidity scarcity. Both components may be increasing in maturity. To illustrate this, we develop in the Appendix two examples based on our theoretical framework. In the first example there is a mismatch between when an asset matures and when an investor may need liquidity. Specifically, if an investor holds a safe asset that matures at  $t = 1$ , she needs no secondary market transaction: the asset pays off exactly when the cash is needed, and its direct liquidation cost is zero. If instead she holds a safe asset that matures at  $t = 2$ , the asset has not yet paid off when the cash is needed, so she must sell it in the secondary market and incur a market access fee to do so. In the second example we model liquidity risk by introducing additional shocks, such that the  $t = 1$  return on the long maturity asset is negatively correlated with the marginal utility of consumption, even if its  $t = 2$  payoff is constant. The short maturity asset in contrast has a fixed  $t = 1$  payoff that does not co-move with these additional shocks.

## **5.2 Measuring relative inconvenience yields: The Treasury-OIS relative spread**

Our empirical exercise focuses on comparing the relative spreads in maturity-matched US Treasury-OIS positions of different tenors. A direct comparison of Treasury yields across maturities cannot separate differences in convenience yields from heterogeneous loading on duration risk. Longer-maturity Treasuries have greater sensitivity to interest rate changes, and their higher yields partly reflect this exposure rather than differences in liquidation costs. We therefore analyse a different relative spread measure.

First, we compute the spreads (or basis) of maturity-matched Treasuries and Overnight indexed swaps (OIS) for different tenors. This step transforms Treasuries of different maturities to instruments with similar duration characteristics, since the OIS component exchanges fixed rates for floating overnight rates, which effectively neutral-

izes the inherent price risk from interest rate changes in longer-dated Treasuries. The Treasury-OIS position spread therefore reflects the cost of holding the position that is not compensation for interest rate risk.<sup>18</sup> In a second step, we take the difference between a specific Treasury-OIS spread and the 3-month Treasury-OIS spread (i.e., a double difference). Formally, for a Treasury-OIS spread of maturity  $M$  we denote this relative spread as  $\text{Relative Spread}_{M-3m}$ . Given the similar duration profile across tenors, the Treasury-OIS relative spread measure therefore compares the relative cost of holding the position of maturity  $M$  relative to the 3-month position.

We use weekly data on Treasury yields and overnight indexed swap (OIS) rates for the period 2010–2023. In different empirical tests we additionally use data on the MOVE Index of implied bond volatility in the US Treasury market (Mallick et al., 2017), and the VIX Index.<sup>19</sup>

### 5.3 Stylized facts

**Fact 1: Treasury-OIS relative spread term structure.** Figure 1 presents the Treasury-OIS relative spread for several maturities, ranging from 1 to 10 years. There is a striking term structure in the Treasury-OIS relative spread – a monotone effect of Treasury-OIS tenor. The effects are also sizable, on the order of around 30 to 40 basis points on average. In addition, whenever the relative spread increases, it increases more for longer maturities.

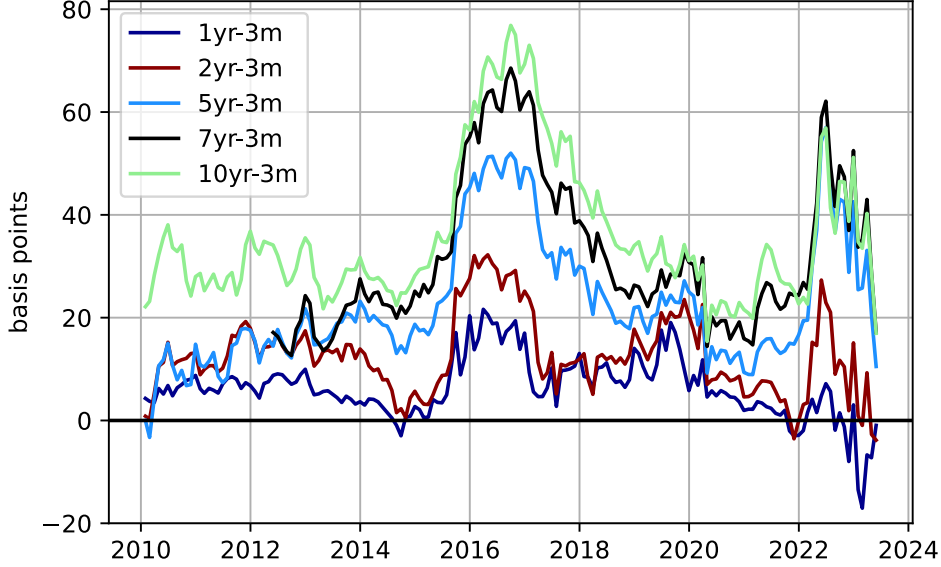
To interpret these findings through the lens of our theory, first note that the longer is the tenor of a Treasury-OIS position, the higher is the probability that the position must be liquidated before it matures with additional costs of reversing the position in the OIS.

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<sup>18</sup>Figure B.1 in the Appendix plots the Treasury-OIS spreads for different maturities. As discussed in the related literature, the Treasury-OIS spread is widely used as a proxy for the convenience yield on Treasuries. In mapping the model to the data, we treat the maturity-matched Treasury-OIS position – not the Treasury security in isolation – as the empirical counterpart of an asset in the model.

<sup>19</sup>We list the various data sources in the Appendix, while Table B.2 presents summary statistics.

Figure 1: Treasury-OIS relative spreads



*Notes:* This figure shows the US Treasury-overnight indexed swap (OIS) spread across different maturities relative to the 3-month Treasury-OIS spread. Source: Bloomberg and authors' calculations.

The Treasury component can be sold in the secondary market, where on-the-run issues are highly liquid even at long maturities. The maturity-increasing cost comes from the swap leg: unwinding a maturity-matched OIS before it expires becomes more costly the longer the remaining life of the contract, both because OIS bid-ask spreads widen with tenor and, more importantly, because a longer-dated swap is harder to cover, since the market for the offsetting contract is thinner and an early exit is less certain to execute at a good price. In contrast, not rolling over 3-month Treasury bills involves less of this structured unwinding – the investor simply declines to roll, receives par, and bears zero liquidation cost. Therefore, the institutional mechanics of the positions we examine imply a transaction cost profile that is increasing in maturity. The term structure from Figure 1 is thus consistent with our Proposition 3, which shows that the relative inconvenience yield  $\psi^i$  is increasing in the asset's transaction cost  $\tau^i$ .

**Fact 2: The MOVE and VIX indices co-move with the Treasury-OIS relative spread.** We regress the Treasury-OIS relative spread for different maturities on the MOVE and VIX indices. To account for effects of the level of interest rates on convenience yields (Nagel (2016), Diamond and Van Tassel (2021)), we also control for the level of interest rates in some specifications by including the effective federal funds rate. We also examine heterogeneous effects of the MOVE and VIX indices for shorter (1 and 2 years) versus longer maturities. Table 1 reports the estimated co-movements.

The Treasury-OIS relative spread correlates positively with the MOVE Index and negatively with the VIX Index. A doubling of the value of the MOVE Index (VIX Index) is associated with an increase (decrease) in the average relative spread of around 8 basis points (column 2). The effects are, however, much weaker for shorter maturity Treasuries (columns 3 and 4).

Table 1: Impact of the MOVE Index and the VIX Index

|                                                     | (1)                   | (2)                   | (3)                    | (4)                    |
|-----------------------------------------------------|-----------------------|-----------------------|------------------------|------------------------|
| $\ln(\text{MOVE})$                                  | 8.9292***<br>(2.416)  | 8.0620***<br>(2.352)  | 14.9371***<br>(3.254)  | 14.0441***<br>(3.212)  |
| $D_{\{1y \text{ or } 2y\}} \times \ln(\text{MOVE})$ |                       |                       | -14.7981***<br>(2.823) | -14.6978***<br>(2.849) |
| $\ln(\text{VIX})$                                   | -8.2040***<br>(2.158) | -7.8005***<br>(2.216) | -13.0780***<br>(2.951) | -12.6975***<br>(3.008) |
| $D_{\{1y \text{ or } 2y\}} \times \ln(\text{VIX})$  |                       |                       | 11.9644***<br>(2.284)  | 12.0007***<br>(2.281)  |
| Fed Funds Rate                                      |                       | 0.7743<br>(0.738)     |                        | 0.7603<br>(0.737)      |
| N                                                   | 3368                  | 3368                  | 3368                   | 3368                   |
| $R^2$                                               | 0.504                 | 0.506                 | 0.521                  | 0.524                  |

Notes: The dependent variable consists of the relative spread  $\text{Relative Spread}_{M-3m}$  of maturity  $M \in \{1y, 2y, 5y, 7y, 10y\}$ . The explanatory variables are the MOVE Index and the VIX Index. In Columns (2) and (4), we additionally control for the Effective Federal Funds Rate.  $D_{\{1y \text{ or } 2y\}}$  is an indicator for the relative spread at 1 or 2 years. All regressions include a constant and maturity fixed effects. The frequency is weekly and the time period is 2010 to mid-May 2023. Standard errors are Driscoll–Kraay heteroskedasticity- and autocorrelation-robust standard errors computed using a Bartlett kernel with bandwidth 5. \*\*\* indicates significance at the 1% level. \*\* indicates significance at the 5% level. \* indicates significance at the 10% level.

We interpret these co-movements as follows. First, the MOVE Index measures the implied volatility of Treasury yields. Higher rate volatility raises the cost of unwinding a duration-neutral Treasury-OIS position before maturity. Part of this is a bond-market effect: bid-ask spreads widen and price impact increases in the Treasury market, which

is the usual reading of MOVE as a proxy for Treasury market illiquidity (Duffie et al., 2023). For the position we study, the cost of covering the swap leg matters as well. When rate uncertainty is high, OIS bid-ask spreads widen and the market gets less liquid, so an early exit from a longer-dated swap contract is more costly and less certain to execute at a good price. Higher values of the MOVE Index can thus be interpreted as a proxy for higher transaction costs.<sup>20</sup> Therefore, the co-movements of the Treasury-OIS relative spread with the MOVE Index are consistent with Proposition 3, namely that an increase in the transaction costs of *any* of the assets increases the relative inconvenience yields for all safe assets. Moreover, the effect is stronger for assets with higher transaction costs, consistent with the estimated interaction effect from Table 1.

VIX measures equity market risk sentiment and therefore proxies for the payoff risk  $\sigma^2$  (or risk aversion parameter  $\gamma$ ) in our model. When VIX rises, investors reallocate from risky to safe assets, and long-term Treasuries appreciate more than short-term ones relative to OIS. The resulting revaluation of safe assets relative to risky assets is exactly what is captured by our Proposition 6, which shows that an increase in payoff risk or risk aversion *reduces* the relative inconvenience yields for all safe assets.

## 6 Conclusion

The interaction of liquidity self-insurance and transaction cost heterogeneity has important asset pricing and welfare implications. Central to that interaction is the value of convenience, an equilibrium pricing factor, which is also a summary statistic for the effect of limited liquidity-risk sharing on agents' welfare. By shedding light on this interaction and the inefficiencies that arise in the competitive equilibrium, our theoretical framework puts in focus the importance of policies that promote and facilitate market liquidity and

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<sup>20</sup>In our model we interpret transaction costs broader than bid-ask spreads. Therefore we consider the MOVE Index as an appropriate proxy measure for these illiquidity costs.

proper market functioning (Duffie, 2023).

Beyond these structural policies, our analysis also points to side-effects of central bank balance sheet policies, via the level of central bank reserves in the system and the asset composition of the central bank’s balance sheet. Our theoretical framework suggests that the long-run volume of central bank reserves in the system may affect aggregate welfare via its influence on liquidity conditions in the economy. It also hints at potentially important aggregate liquidity effects of the maturity structure as well as the taxation regime of government debt. Nevertheless, our simple theory includes only one channel through which these policy tools impact aggregate welfare. Understanding the full set of complex trade-offs and the resulting optimal policy mix is an important avenue for future research on this topic.

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# A Theory Appendix

## A.1 Additional results

### A.1.1 Buyer transaction subsidies

A suitable set of buyer subsidies for different assets can restore full efficiency in the competitive equilibrium. We assume that the expenditure is financed with period  $t = 1$  lump-sum taxes. Also, to streamline the exposition, we assume that the transaction costs are sufficiently small, so that it is feasible to implement the full set of buyer subsidies that can restore optimality. Specifically, we can show the following result:

**Proposition 10 (Buyer subsidization).** *Suppose that  $\tau_b^i = -\tau^i, \forall i$ . Then the economy achieves full liquidity risk-sharing, with  $\eta = 1$  in equilibrium.*

*Proof.* See Appendix A.2. □

The intuition for this result is as follows. Given perfectly competitive asset markets at  $t = 1$  and perfectly elastic asset demands by buyers, any subsidy that a buyer receives when purchasing a specific asset is fully passed through into the  $t = 1$  asset price for that asset. Therefore, the buyer does not benefit from the subsidy in equilibrium, as she only bids up the asset price. This increase in the asset price, however, compensates the seller for incurring the transaction cost when selling that asset. By setting the buyer subsidy appropriately, one can manipulate the equilibrium asset prices in a way that negates the effect of the seller transaction costs, thus achieving the first-best level of liquidity risk-sharing in the competitive equilibrium.

### A.1.2 Relating transaction costs and asset maturity: micro-foundations

We provide two micro-founded examples for higher transaction costs on longer maturity assets. For both examples we adapt our framework by assuming, for simplicity, that there

are only two risk-free assets. We further modify the framework slightly by assuming that one asset matures in period  $t = 1$ , while the other matures in  $t = 2$ , which introduces a term structure into our model.

In the first example we additionally assume that if the two-period asset has to be resold at  $t = 1$ , the investor has to pay a fee, which can be thought of as a market access fee. Our results show that the long maturity asset is less convenient and therefore may be associated with higher transaction costs. Unlike the short maturity asset, which pays off precisely when an investor has liquidity needs, the long maturity asset has to be liquidated, while incurring a cost to do so.

In the second example we do not assume that agents incur any transaction costs when selling assets. In contrast, we focus on the impact of aggregate uncertainty on the convenience of assets with different maturities. As a concrete example (which we later generalize) we assume that the endowment in  $t = 1$  is stochastic. There is a news shock about the aggregate endowment realized in  $t = 1$ , so that at  $t = 0$  agents do not know if their endowment will be high or low in the second period. Our results reveal that the price, and hence the return, of the long maturity asset in period  $t = 1$  covaries negatively with the marginal utility of consumption. In comparison, the short maturity asset matures in  $t = 1$  and therefore its payoff is not correlated with endowment uncertainty. The negative covariance between the future return and future marginal utility (and therefore also with the aggregate value of convenience) is another reason why the long maturity asset is less convenient to be held at  $t = 0$  and trades at a discount relative to the short maturity asset.

We also generalize these examples by deriving a general expression for the relative inconvenience yield within our specialized set-up with two assets with different maturities. We assume both market fees and aggregate uncertainty, thus combining the two examples. Compared to the second example we further assume that the aggregate uncertainty

can apply to any exogenous variable, for example payoffs. Additionally, we assume an isoelastic utility function instead of a log utility function to generalize further. We also assume that the third period is discounted with a discount factor that is possibly smaller than one (to allow for aggregate uncertainty on the discount factor). We show that the expression for the relative inconvenience yield nests our two examples from before, and that the drivers are the same. With respect to the influence of aggregate uncertainty, it depends on the specific kind of uncertainty whether the long or the short maturity asset is more convenient.

**Preliminaries** We use our original model and make the following adjustment to explicitly account for short and long maturity assets: We assume (for simplicity) that there are only two risk-free assets in the economy. One matures after one period and the other after two periods. We denote the short maturity asset by  $l$  (“low”) and the long maturity asset by  $h$  (“high”). We will give two examples of why the long maturity asset is less convenient relative to the short maturity asset.

**Example 1** First we assume that if an asset is sold before maturing, the agents incur a fee. As this can only happen for the long maturity asset, the fee is denoted by  $\tau^h$ . We start by describing the optimal behavior in period 1. As in our standard model, the liquidity-constrained agent will liquidate all assets in period 1 and therefore  $X_{2,\bar{S}}^l = X_{2,\bar{S}}^h = 0$ . The other kind of agent solves  $\max_{X_{2,\underline{S}}^h} \log(Y_1 + A_1 - P_1^h X_{2,\underline{S}}^h) + \log(\delta^h X_{2,\underline{S}}^h)$ , where  $A_1 \equiv \delta^l X_1^l + P_1^h X_1^h$ . The first-order condition is given by  $\frac{P_1^h}{Y_1 + A_1 - P_1^h X_{2,\underline{S}}^h} = \frac{1}{X_{2,\underline{S}}^h}$ , or  $P_1^h X_{2,\underline{S}}^h = \frac{Y_1 + A_1}{2}$ . Market clearing implies  $\lambda(X_{2,\bar{S}}^h - X_1^h) + (1 - \lambda)(X_{2,\underline{S}}^h - X_1^h) = 0$ , or

$X_{2,\underline{S}}^h = \frac{X_1^h}{(1-\lambda)}$ . In period 0 the agents solve the following maximization problem:

$$\max_{c_0, X_1^l, X_1^h} \log(c_0) + \lambda \chi V_{1,\bar{S}} + (1-\lambda) V_{1,\underline{S}}$$

s.t.

$$Y_0 = c_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)$$

where  $V_{1,\bar{S}} = \log(Y_1 + A_1 - \tau^h P_1^h X_1^h)$  and  $V_{1,\underline{S}} = \log\left(Y_1 + A_1 - \frac{P_1^h X_1^h}{1-\lambda}\right) + \log\left(\frac{\delta^h X_1^h}{1-\lambda}\right)$ , or  $V_{1,\underline{S}} = \log\left[Y_1 + A_1 - \frac{Y_1 + A_1}{2}\right] + \log\left[\frac{\delta^h}{P_1^h} \frac{Y_1 + A_1}{2}\right]$ . The first-order conditions are given by

$$\frac{1}{Y_0 - P_0^l(Q^l - X_1^l) - P_0^h(Q^h - X_1^h)} = \lambda \left( \frac{\chi}{Y_1 + A_1 - \tau^h X_1^h} \frac{\delta^l}{P_0^l} \right) + (1-\lambda) \left( \frac{2}{Y_1 + A_1} \frac{\delta^l}{P_0^l} \right)$$

and

$$\frac{1}{Y_0 - P_0^l(Q^l - X_1^l) - P_0^h(Q^h - X_1^h)} = \lambda \left( \frac{\chi(1-\tau^h)}{Y_1 + A_1 - \tau^h X_1^h} \frac{P_1^h}{P_0^h} \right) + (1-\lambda) \left( \frac{2}{Y_1 + A_1} \frac{P_1^h}{P_0^h} \right).$$

Market clearing in period 0 implies that  $X_1^l = Q^l$  and  $X_1^h = Q^h$ . From combining the first-order condition of the liquidity unconstrained agent in period 1 and market clearing, it follows that  $P_1^h = \frac{1-\lambda}{1+\lambda} \frac{Y_1 + \bar{Q}^l}{Q^h}$ . From now on we assume that  $\chi = 2$  (as in the main body). In addition we define  $R_1^h \equiv \frac{P_1^h}{P_0^h}$  and  $R_1^l \equiv \frac{\delta^l}{P_0^l}$ . Note that  $\frac{\partial V_{1,s}}{\partial A_1} = \frac{\partial U_{1,s}}{\partial c_{1,s}}$ , and rewriting the first of the two first-order conditions implies  $\frac{1}{R_1^l} = E_s \left( \frac{\frac{\partial U_{1,s}}{\partial c_{1,s}}}{\frac{\partial U_1}{\partial c_0}} \right)$ . As a next step we combine the first-order conditions in period 1. To rearrange we use

$E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} \right) \equiv \lambda \left( \frac{\partial U_{1,\bar{s}}}{\partial c_{1,\bar{s}}} \right) + (1 - \lambda) \left( \frac{\partial U_{1,\underline{s}}}{\partial c_{1,\underline{s}}} \right)$ . Rearranging yields

$$\begin{aligned}
0 &= \lambda \frac{2}{Y_1 + A_1 - \tau^h Q^h} [R_1^l - R_1^h + R_1^h \tau^h] + (1 - \lambda) \left[ \frac{2}{Y_1 + A_1} (R_1^l - R_1^h) \right] \\
0 &= E_s \left\{ \frac{\partial U_{1,s}}{\partial c_{1,s}} \left[ 1 - \frac{R_1^h}{R_1^l} (1 + \mathcal{I}_{s=\bar{s}} \tau^h) \right] \right\} \\
0 &= COV \left[ \frac{\partial U_{1,s}}{\partial c_{1,s}}, \mathcal{I}_{s=\bar{s}} \tau^h \right] \frac{R_1^h}{R_1^l} + E_s \left[ \frac{\partial U_{1,s}}{\partial c_{1,s}} \right] \left[ 1 - \frac{R_1^h}{R_1^l} (1 - \lambda \tau^h) \right] \\
0 &= COV [m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^h] \frac{R_1^h}{R_1^l} + E_s [m_{1,s}] \left[ 1 - \frac{R_1^h}{R_1^l} (1 - \lambda \tau^h) \right] \\
0 &= \frac{1}{R_1^l} + [COV [m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^h] - E_s [m_{1,s}] (1 - \lambda \tau^h)] \frac{R_1^h}{R_1^l}.
\end{aligned}$$

Further rearranging yields

$$\frac{R_1^h}{R_1^l} = \frac{1}{R_1^l} \frac{1}{\{E_s [m_{1,s}] (1 - \lambda \tau^h) - COV [m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^h]\}}.$$

We know that  $COV [m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^h] = \lambda \tau^h m_{1,\bar{s}} - E_s [m_{1,s}] \lambda \tau^h$ . Therefore

$$\begin{aligned}
\frac{R_1^h}{R_1^l} &= \frac{1}{1 - \lambda \tau^h - \lambda \tau^h [m_{1,\bar{s}} - E_s [m_{1,s}]] R_1^l} > 0 \\
\frac{R_1^h}{R_1^l} &= \frac{1}{1 - \lambda \tau^h - \lambda \tau^h (1 - \lambda)(\eta - 1) m_{1,\underline{s}} R_1^l} > 0.
\end{aligned}$$

The long maturity asset is less convenient. The short maturity asset carries a convenience premium when being compared to the long maturity asset. The reason is that the long maturity asset has a lower probability to mature at the moment the liquidity is needed.

**Example 2** Second, we assume that there is aggregate uncertainty in  $t = 1$ . We assume that the endowment can be either high or low in the first period. In period 0, the value of the endowment, denoted by  $Y_1^k$ , is unknown, but the agents know the distribution. We use this explicit example for illustrative purposes and later generalize the micro-foundation.

Analogous to the above example and adjusted to the current example, the first-order

conditions are given by

$$\frac{1}{Y_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)} = E \left( \frac{2}{Y_1^k + A_1} \frac{\delta^l}{P_0^l} \right)$$

and

$$\frac{1}{Y_0 + P_0^l(Q^l - X_1^l) + P_0^h(Q^h - X_1^h)} = E \left( \frac{2}{Y_1^k + A_1} \frac{P_1^h}{P_0^h} \right).$$

We combine the first-order conditions. Note that  $\frac{\partial U_{1,\underline{S}}}{\partial A_1} = \frac{\partial U_{1,\bar{S}}}{\partial A_1}$  and  $m_{1,\underline{S}} = m_{1,\bar{S}}$ . We will use  $\frac{1}{R_1^l} = E \left( \frac{\frac{\partial U_{1,\underline{S}}}{\partial A_1}}{\frac{\partial U_1}{\partial c_0}} \right)$ . Therefore

$$\begin{aligned} 0 &= E \left[ \frac{2}{Y_1^k + A_1} (R_1^l - R_1^h) \right] \\ 0 &= E \left[ \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \left( 1 - \frac{R_1^h}{R_1^l} \right) \right] \\ 0 &= E \left[ m_{1,\underline{S}} \left( 1 - \frac{R_1^h}{R_1^l} \right) \right] \\ 0 &= \frac{1}{R_1^l} - E \left( m_{1,\underline{S}} \frac{R_1^h}{R_1^l} \right) \\ 0 &= \frac{1}{R_1^l} - COV \left( \frac{R_1^h}{R_1^l}, m_{1,\underline{S}} \right) - E \left( \frac{R_1^h}{R_1^l} \right) E(m_{1,\underline{S}}). \end{aligned}$$

Further rearranging yields

$$\begin{aligned} E \left( \frac{R_1^h}{R_1^l} \right) &= \frac{\frac{1}{R_1^l} - COV \left( m_{1,\underline{S}}, \frac{R_1^h}{R_1^l} \right)}{E(m_{1,\underline{S}})} \\ E \left( \frac{R_1^h}{R_1^l} \right) &= 1 - COV(m_{1,\underline{S}}, R_1^h) \\ E \left( \frac{R_1^h}{R_1^l} \right) &= 1 - COV \left( \frac{m_{1,\bar{S}}}{\eta}, R_1^h \right) \\ E \left( \frac{R_1^h}{R_1^l} \right) &= 1 - 2 COV \left( \frac{Y_0}{Y_1^k + A_1}, R_1^h \right). \end{aligned}$$

From  $P_1^h = \frac{1-\lambda}{1+\lambda} \frac{Y_1^k + \bar{Q}^l}{Q^h}$  it follows that  $\frac{\partial P_1^h}{\partial Y_1^k} > 0$ . Therefore  $COV \left[ \frac{Y_0}{Y_1^k + A_1}, R_1^h \right] < 0$ . It

follows that

$$E \left( \frac{R_1^h}{R_1^l} \right) > 1.$$

The long maturity asset is on average less convenient because in period  $t = 1$  the asset has a high price or return when the marginal utility is low.

**Generalization** Finally, we can derive (following the same steps as in the examples above) a more general function that incorporates both channels, transaction costs and aggregate uncertainty, and further generalizes the latter. First, we generalize by using an isoelastic utility function instead of a log utility function. Second, we allow agents to discount the second period with the factor  $\beta \leq 1$ . Third, we now leave open which variable  $x^i$  is affected by an aggregate uncertainty in period 0 that is revealed in period 1. For example, we could introduce payoff risk as an aggregate risk or an uncertainty in the discount factor  $\beta$ . The aggregate uncertainty in the endowment was just an example.

The result that the covariance and not any volatility implies the effect of aggregate uncertainty on the relative convenience return is more general. We derive a general function for the relative inconvenience yields (which nests the relative inconvenience yields of the above examples):

$$E \left( \frac{R_1^h}{R_1^l} \right) = \frac{1 + COV [COV [m_{1,s}, \mathcal{I}_{s=\bar{S}} \tau^h] - E_s [m_{1,s}] (1 - \lambda \tau^h), R_1^h]}{1 - \lambda \tau^h - E \{ COV [m_{1,s}, \mathcal{I}_{s=\bar{S}} \tau^h] R_1^l \}},$$

or

$$\begin{aligned} E \left( \frac{R_1^h}{R_1^l} \right) &= \frac{1 + COV [\lambda \tau^h m_{1,\bar{S}} - E_s [m_{1,s}], R_1^h]}{1 - \lambda \tau^h - \lambda \tau^h E [m_{1,\bar{S}} - E_s [m_{1,s}]] R_1^l} \\ E \left( \frac{R_1^h}{R_1^l} \right) &= \frac{1 + COV [\lambda (\tau^h - 1) \eta - (1 - \lambda)] m_{1,\underline{S}}, R_1^h]}{1 - \lambda \tau^h - \lambda \tau^h E [(1 - \lambda)(\eta - 1) m_{1,\underline{S}}] R_1^l}. \end{aligned}$$

## A.2 Omitted proofs

### Proof of Lemma 1

*Proof.* We describe and solve the optimization problems in the second and third period.

**$t = 1$  problem of patient investors** For the problem of the patient investor in period 1, we split the problem into a portfolio-choice problem and a consumption-saving problem. Specifically, we conjecture that in equilibrium she only increases her position in each asset. We then split her problem into two: a consumption-saving decision (problem 1) given by

$$\begin{aligned} V_{1,\underline{S}} &= \max_{c_{1,\underline{S}}, A_{2,\underline{S}}} \log(c_{1,\underline{S}}) + \log(V_2(A_{2,\underline{S}})) \\ &\text{s.t.} \\ c_{1,\underline{S}} + A_{2,\underline{S}} &= Y_1 + A_1, \end{aligned}$$

where  $A_{2,\underline{S}}$  denotes savings into  $t = 2$ , and a portfolio-choice problem (problem 2), given by

$$\begin{aligned} V_2(A_{2,\underline{S}}) &= \max_{X_{2,\underline{S}}^f, X_{2,\underline{S}}^m} \left( E \left[ c_{2,\underline{S}}(z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)} \\ &\text{s.t.} \\ \sum_{i \in \mathbb{I}} P_1^i X_{2,\underline{S}}^i + P_1^m X_{2,\underline{S}}^m &= A_{2,\underline{S}} \\ c_{2,\underline{S}}(z) &= \sum_{i \in \mathbb{I}} \delta^i X_{2,\underline{S}}^i + \varphi^m(z) X_{2,\underline{S}}^m, \quad \forall z. \end{aligned}$$

Note that the solution to this problem can be fully summarized by the portfolio share invested in the risky asset, namely  $w \equiv \frac{P_1^m X_{2,\underline{S}}^m}{A_{2,\underline{S}}}$ . In equilibrium, that portfolio share will satisfy  $0 \leq w \leq 1$ . Therefore, the portfolio problem can equivalently be written as

$$\begin{aligned}
V_2(A_{2,\underline{S}}) &= \max_w \left( E \left[ c_{2,\underline{S}}(z)^{(1-\gamma)} \right] \right)^{1/(1-\gamma)} \\
&\text{s.t.} \\
c_{2,\underline{S}}(z) &= \left[ (1-w) \frac{1}{\bar{P}_1^f} + w \frac{\varphi^m(z)}{P_1^m} \right] A_{2,\underline{S}}, \quad \forall z.
\end{aligned}$$

We used  $P_1^f = \frac{P_1^i}{\delta^i}$  which we demonstrate in Lemma 3 below.

One can then compute the asset allocations as  $\sum_{i \in \mathbb{I}} P_1^i X_{2,\underline{S}}^i = (1-w) A_{2,\underline{S}}$ , and  $P_1^m X_{2,\underline{S}}^m = w A_{2,\underline{S}}$ . Given homothetic preferences, Lemma 4 below shows that the value function  $V_2(A_{2,\underline{S}})$  is linear in savings, i.e.,

$$V_2(A_{2,\underline{S}}) = R_2^{CE} A_{2,\underline{S}}$$

and the optimal value of  $w$  satisfies

$$E \left[ \left[ \frac{1}{\bar{P}_1^f} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w \right]^{-\gamma} \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) \right] = 0.$$

The period  $t = 1$  consumption-saving problem becomes

$$\begin{aligned}
V_{1,\underline{S}} &= \max_{c_{1,\underline{S}}, A_{2,\underline{S}}} \log(c_{1,\underline{S}}) + \log(R_2^{CE} A_{2,\underline{S}}) \\
&\text{s.t.} \\
c_{1,\underline{S}} + A_{2,\underline{S}} &= Y_1 + A_1.
\end{aligned}$$

With log utility, we then have  $A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}$  and  $c_{1,\underline{S}} = \frac{Y_1 + A_1}{2}$ . We insert the optimal values into the period  $t = 1$  utility function  $U_{1,\underline{S}}$  and derive the period  $t = 1$  value function,  $V_{1,\underline{S}}$ , which is given by  $V_{1,\underline{S}} = \log R_2^{CE} \left( \frac{Y_1 + A_1}{2} \right)^2$ .

**$t = 1$  problem of impatient investors** The problem of the impatient investor in period 1 is trivial as she sells all her asset holdings and consumes all available resources at  $t = 1$ . Therefore,

$$X_{2,\bar{S}}^i = 0 \quad \forall i$$

$$X_{2,\bar{S}}^m = 0$$

and we have  $c_{1,\bar{S}} = Y_1 + A_1 - \sum_i \tau^i P_1^i Q^i - \tau^m P_1^m Q^m$ , where  $A_1 \equiv \sum_i P_1^i Q^i + P_1^m Q^m$  denotes the investor's financial wealth at  $t = 1$ .

**Market clearing at  $t = 1$**  At  $t = 1$  the patient and impatient investors trade assets with one another. Taking into account the respective mass of each type of investor and their asset holding decisions, market clearing conditions are given by

$$(1 - \lambda)X_{2,\underline{S}}^i = Q^i, \quad \forall i \in \mathbb{I}$$

$$(1 - \lambda)X_{2,\underline{S}}^m = Q^m.$$

Therefore, patient investors hold all the assets at the end of  $t = 1$ . From these market clearing conditions we can derive the period 1 prices as well as the equilibrium value of period  $t = 1$  financial wealth and consumption. Using the market clearing conditions and  $A_{2,\underline{S}} = \frac{Y_1 + A_1}{2}$  from the consumption-saving decision and the optimal portfolio weight  $w$  on the risky asset, we end up with the following equation for the risky asset:

$$\frac{1}{(1 - \lambda)} P_1^m Q^m = w \frac{Y_1 + A_1}{2}. \quad (\text{A.1})$$

and

$$\frac{1}{(1 - \lambda)} \bar{P}_1^f \bar{Q}^f = (1 - w) \frac{Y_1 + A_1}{2} \quad (\text{A.2})$$

for the risk-free assets. Summing these two equations yields

$$\frac{1}{(1-\lambda)}P_1^m Q^m + \frac{1}{(1-\lambda)}\bar{P}_1^f \bar{Q}^f = \frac{A_1}{1-\lambda} = \frac{Y_1 + A_1}{2},$$

which further yields the equilibrium value of period  $t = 1$  asset holdings of

$$A_1 = \frac{(1-\lambda)}{(1+\lambda)}Y_1. \quad (\text{A.3})$$

Substituting for  $A_1$  into Eqs. (A.4) and (A.5), we arrive at the equilibrium pricing equations

$$P_1^m Q^m = w \frac{(1-\lambda)}{(1+\lambda)}Y_1, \quad (\text{A.4})$$

and

$$\bar{P}_1^f \bar{Q}^f = (1-w) \frac{(1-\lambda)}{(1+\lambda)}Y_1. \quad (\text{A.5})$$

Lastly, the budget constraints in combination with the optimal asset holdings yield the consumption choices in all periods:

$$c_{1,\bar{S}} = \frac{2}{1+\lambda}Y_1 - \sum_i \tau^i P_1^i Q^i - \tau^m P_1^m Q^m$$

$$c_{1,\underline{S}} = \frac{1}{1+\lambda}Y_1.$$

□

## Proof of Lemma 2

*Proof.* We describe and solve the first period optimization problem.

**$t = 0$  problem of investors** The period  $t = 0$  problem of the representative investor is given, recursively, by

$$\begin{aligned} V_0 &= \max_{c_0, \{X_1^i\}_{i \in \mathbb{I}}, X_1^m} \log(c_0) + \lambda V_{1,\bar{S}} + (1 - \lambda) V_{1,\underline{S}} \\ \text{s.t.} \\ c_0 &= Y_0 + \sum_{i \in \mathbb{I}} P_0^i (Q^i - X_1^i) + P_0^m (Q^m - X_1^m), \end{aligned}$$

where  $V_{1,\bar{S}} = 2 \log(c_{1,\bar{S}})$ , with  $c_{1,\bar{S}} = Y_1 + A_1 - \sum_{i \in \mathbb{I}} \tau^i P_1^i X_1^i - \tau^m P_1^m X_1^m$  and  $V_{1,\underline{S}} = \log R_2^{CE}(c_{1,\underline{S}})^2$  with  $c_{1,\underline{S}} = \frac{Y_1 + A_1}{2}$ . The Euler equation for holdings of asset  $i \in \mathbb{I} \cup \{m\}$  is

$$\frac{\partial U_0}{\partial c_0} = E_s \left[ \frac{\partial V_{1,s}}{\partial A_1} \frac{P_1^i}{P_0^i} (1 - \mathcal{I}_{s=\bar{S}} \tau^i) \right],$$

or

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}} \frac{P_1^i}{P_0^i} (1 - \tau^i) + (1 - \lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^i}{P_0^i},$$

where we have used the fact that at  $t = 1$ , by the Envelope Theorem for the agent's  $t = 1$  problem,  $\frac{\partial V_{1,s}}{\partial A_1} = \frac{\partial U_{1,s}}{\partial c_{1,s}}$ . Therefore, at the optimum, the marginal utility of consumption today equals the expected marginal utility of consumption tomorrow, where the expectation is taken over the realization of the idiosyncratic investor's state  $s$  and the transaction cost incurred in the impatient state is taken into account.

**Market clearing and consumption at  $t = 0$**  Market clearing at  $t = 0$  is given by

$$\begin{aligned} X_1^i &= Q^i, \quad \forall i \\ X_1^m &= Q^m. \end{aligned}$$

Substituting into the period budget constraint, we in turn have  $c_0 = Y_0$ . □

Additionally, we show the following two Lemmas, which constitute parts of the proof of Lemma 1.

**Lemma 3 (Second period prices).**  $P_1^f = \frac{P_1^i}{\delta^i}, \forall i \in \mathbb{I}$ .

*Proof.* The Lagrangian of the portfolio-choice problem of the patient investor in period 1 is given by

$$\begin{aligned} \mathcal{L} = & \left( \int_z \left[ \left( \sum_i \delta^i X_{2,\underline{S}}^i + R_2^m(z) X_{2,\underline{S}}^m \right)^{(1-\gamma)} \right] d(z) \right)^{\frac{1}{(1-\gamma)}} \\ & + \mu \left[ A_{2,\underline{S}} - \sum_i P_1^i X_{2,\underline{S}}^i - P_1^m X_{2,\underline{S}}^m \right]. \end{aligned}$$

The first-order conditions with respect to the risk-free assets (given an interior solution) are

$$\begin{aligned} & \frac{1}{(1-\gamma)} \left( \int_z \left[ \left( \sum_i \delta^i X_{2,\underline{S}}^i + R_2^m(z) X_{2,\underline{S}}^m \right)^{(1-\gamma)} \right] dF(z) \right)^{\frac{1}{(1-\gamma)}-1} \\ & = \frac{\mu P_1^i}{(1-\gamma)\delta^i \left[ \int_z \left( \sum_i \delta^i X_{2,\underline{S}}^i + R_2^m(z) X_{2,\underline{S}}^m \right)^{(-\gamma)} dF(z) \right]} \end{aligned}$$

for all  $i \in \mathbb{I}$ . We define  $\bar{P}_1^i \equiv \frac{P_1^i}{\delta^i}$ . It follows that  $\bar{P}_1^i$  is the same for all  $i \in \mathbb{I}$ . We denote this price by  $\bar{P}_1^f$ . □

**Lemma 4 (Third period utility).**  $V_2(A_{2,\underline{S}}) = R_2^{CE} A_{2,\underline{S}}$ , and  $w$  satisfies

$$E \left[ \left[ \frac{1}{\bar{P}_1^f} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w \right]^{-\gamma} \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) \right] = 0.$$

*Proof.* Using  $\bar{P}_f^1 = \frac{P_1^1}{\delta^1}$  for all  $i$ , the portfolio-choice problem of the patient investor in

period 1 can be rewritten as

$$\max_w \left[ \int_z \left[ \frac{1}{\bar{P}_1^f} A_{2,\underline{S}} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w A_{2,\underline{S}} \right]^{(1-\gamma)} dF(z) \right]^{\frac{1}{(1-\gamma)}}.$$

The first-order condition for  $w$  is given by

$$[V_2(A_{2,\underline{S}})]^\gamma \int_z \left[ \frac{1}{\bar{P}_1^f} A_{2,\underline{S}} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w A_{2,\underline{S}} \right]^{-\gamma} \left[ \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) A_{2,\underline{S}} \right] dF(z) = 0.$$

We can rewrite as

$$[V_2(A_{2,\underline{S}})]^\gamma A_{2,\underline{S}}^{1-\gamma} \int_z \left[ \frac{1}{\bar{P}_1^f} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w \right]^{-\gamma} \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) dF(z) = 0.$$

Note that this condition holds for any value of  $A_{2,\underline{S}}$ . Therefore, the first-order condition for  $w$  simplifies to

$$\int_z \left[ \frac{1}{\bar{P}_1^f} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w \right]^{-\gamma} \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) dF(z) = 0, \quad (\text{A.6})$$

so that the optimal choice of  $w$  is independent of the value of  $A_{2,\underline{S}}$ . We can, therefore, write

$$V_2(A_{2,\underline{S}}) = R_2^{CE} A_{2,\underline{S}},$$

where

$$R_2^{CE} \equiv \left[ \int_z \left[ \frac{1}{\bar{P}_1^f} + \left( \frac{\varphi^m(z)}{P_1^m} - \frac{1}{\bar{P}_1^f} \right) w \right]^{(1-\gamma)} dF(z) \right]^{\frac{1}{(1-\gamma)}},$$

where  $w$  solves the first-order condition, Eq. (A.6).

□

## Proof of Proposition 2

*Proof.* We know that the first-order conditions of the period  $t = 0$  problem for all assets are

$$\frac{1}{c_0} = E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} \frac{P_1^i (1 - \mathcal{I}_{s=\bar{S}} \tau^i)}{P_0^i} \right)$$

for all  $i \in \mathbb{I} \cup m$ . Let us define  $R_{1,s}^i \equiv \frac{P_1^i (1 - \mathcal{I}_{s=\bar{S}} \tau^i)}{P_0^i}$ ,  $R_1^1 \equiv \frac{P_1^1}{P_0^1}$ , and  $m_{1,s} \equiv \frac{\frac{\partial U_{1,s}}{\partial c_{1,s}}}{\frac{\partial U_0}{\partial c_0}}$ . Then  $\frac{\partial U_0}{\partial c_0} = E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} R_{1,s}^i \right)$  for all  $i$ . This means that  $\frac{\partial U_0}{\partial c_0} = R_1^1 E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} \right)$  must also hold. We rearrange the general formula as follows:

$$\begin{aligned} 0 &= E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} R_{1,s}^i - \frac{\partial U_0}{\partial c_0} \right) \\ 0 &= E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} (R_{1,s}^i - R_1^1) \right) \\ 0 &= E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} \right) E_s (R_{1,s}^i - R_1^1) + COV \left( \frac{\partial U_{1,s}}{\partial c_{1,s}}, (R_{1,s}^i - R_1^1) \right) \\ - \frac{COV \left( \frac{\partial U_{1,s}}{\partial c_{1,s}}, (R_{1,s}^i - R_1^1) \right)}{E_s \left( \frac{\partial U_{1,s}}{\partial c_{1,s}} \right)} &= E_s (R_{1,s}^i) - R_1^1 \\ -COV (m_{1,s}, (R_{1,s}^i - R_1^1)) R_1^1 &= E_s (R_{1,s}^i) - R_1^1 \\ -COV (m_{1,s}, R_{1,s}^i) &= \frac{E_s (R_{1,s}^i)}{R_1^1} - 1. \end{aligned}$$

We define the relative inconvenience yield  $\psi^i \equiv \frac{R_{1,s}^i}{R_1^1}$  with  $R_1^1 \equiv \frac{P_1^1}{P_0^1}$ . We insert for  $R_{1,s}^i$  and rearrange:

$$\begin{aligned} \frac{E_s \left( \frac{P_1^i (1 - \mathcal{I}_{s=\bar{S}} \tau^i)}{P_0^i} \right)}{R_1^1} - 1 &= -COV \left( m_{1,s}, \frac{P_1^i (1 - \mathcal{I}_{s=\bar{S}} \tau^i)}{P_0^i} \right) \\ \frac{\frac{P_1^i}{P_0^i}}{R_1^1} (1 - \lambda \tau^i) - 1 &= -\frac{\frac{P_1^i}{P_0^i}}{R_1^1} COV (m_{1,s}, (1 - \mathcal{I}_{s=\bar{S}} \tau^i)) R_1^1 \\ \psi^i &= \frac{1}{(1 - \lambda \tau^i) - COV (m_{1,s}, \mathcal{I}_{s=\bar{S}} \tau^i) R_1^1}. \end{aligned}$$

We then rearrange the covariance term:

$$\begin{aligned} COV(m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^i) &= \lambda \tau^i m_{1,\bar{s}} - E_s(m_{1,s}) \lambda \tau^i \\ COV(m_{1,s}, \mathcal{I}_{s=\bar{s}} \tau^i) &= \lambda \tau^i \left( m_{1,\bar{s}} - \frac{1}{R_1^1} \right). \end{aligned}$$

We insert it into our main term of interest:

$$\begin{aligned} \psi^i &= \frac{1}{1 - \lambda \tau^i m_{1,\bar{s}} R_1^1} \\ \psi^i &= \frac{1}{1 - \lambda \tau^i m_{1,\bar{s}} \frac{1}{E_s(m_{1,s})}} \\ \psi^i &= \frac{1}{1 - \tau^i \frac{\lambda m_{1,\bar{s}}}{\lambda m_{1,\bar{s}} + (1-\lambda) m_{1,\underline{s}}}}. \end{aligned}$$

Therefore the relative inconvenience yield is given by

$$\psi^i = \frac{1}{1 - \tau^i \frac{\lambda}{\lambda + (1-\lambda)/\eta}}$$

where  $\eta \equiv \frac{m_{1,\bar{s}}}{m_{1,\underline{s}}}$  is the relative kernel, which we later call the value of convenience.  $\square$

### Proof of Proposition 3

*Proof.* We analyze the impact of the fees on the relative inconvenience yields. Define

$$\zeta \equiv \frac{1 - \lambda}{\lambda + (1 - \lambda)/\eta}.$$

By Proposition 2 the derivatives are given by

$$\frac{\partial \psi^i}{\partial \tau^i} = -(\psi^i)^2 \frac{\lambda}{\lambda + (1 - \lambda)/\eta} \left( \zeta \tau^i \frac{\partial(1/\eta)}{\partial \tau^i} - 1 \right) > 0$$

and

$$\frac{\partial \psi^i}{\partial \tau^k} = \frac{\partial \psi^i}{\partial (1/\eta)} \frac{\partial (1/\eta)}{\partial \tau^k} > 0 \text{ for } k \neq i \text{ and for all } i \in (\mathbb{I} \setminus \{1\}) \cup m,$$

where

$$\frac{\partial \psi^i}{\partial (1/\eta)} = -(\psi^i)^2 \frac{\tau^i \lambda (1 - \lambda)}{(\lambda + (1 - \lambda)/\eta)^2} < 0,$$

and, by Eq. (11),

$$\frac{\partial (1/\eta)}{\partial \tau^j} = \begin{cases} -\frac{1-\lambda}{2}(1-w)\frac{\bar{Q}^j}{\bar{Q}^f} < 0, & j \in (\mathbb{I} \setminus \{1\}), \\ -\frac{1-\lambda}{2}w < 0, & j = m. \end{cases}$$

In addition it follows that

$$\begin{aligned} \frac{\partial^2 \psi^i}{\partial (\tau^i)^2} = & -2\psi^i \frac{\partial \psi^i}{\partial \tau^i} \frac{\lambda}{\lambda + (1 - \lambda)/\eta} \left( \zeta \tau^i \frac{\partial (1/\eta)}{\partial \tau^i} - 1 \right) + (\psi^i)^2 \frac{\lambda}{\lambda + (1 - \lambda)/\eta} \left[ -\zeta \frac{\partial (1/\eta)}{\partial \tau^i} \right. \\ & \left. + \zeta \frac{\partial (1/\eta)}{\partial \tau^i} \left( \zeta \tau^i \frac{\partial (1/\eta)}{\partial \tau^i} - 1 \right) + \zeta^2 \tau^i \left( \frac{\partial (1/\eta)}{\partial \tau^i} \right)^2 \right] > 0. \end{aligned}$$

and

$$\frac{\partial^2 \psi^i}{\partial \tau^k \partial \tau^i} = -\frac{\lambda(1 - \lambda)}{(\lambda + (1 - \lambda)/\eta)^2} \left[ 2\psi^i \frac{\partial \psi^i}{\partial \tau^i} \tau^i + (\psi^i)^2 \left( 1 + \tau^i(1 - \lambda) \frac{\partial (1/\eta)}{\partial \tau^i} \right) \right] \frac{\partial (1/\eta)}{\partial \tau^k} > 0.$$

□

#### Proof of Proposition 4

*Proof.* This change leaves total payoff-adjusted risk-free asset supply unchanged, i.e.

$$d\bar{Q}^f = d\bar{Q}^l + d\bar{Q}^k = 0.$$

The equilibrium portfolio share  $w$  depends on the total supplies  $\bar{Q}^f$  and  $Q^m$ , but not on the composition of  $\bar{Q}^f$ . Consequently,  $dw = 0$ . Since  $\bar{Q}^f$  is constant, the change in the average transaction cost is

$$\frac{d\bar{\tau}}{d\bar{Q}^l} = \frac{\tau^l - \tau^k}{\bar{Q}^f}.$$

From Eq. (11) it follows that

$$\frac{d(1/\eta)}{d\bar{Q}^l} = \frac{1-\lambda}{2}(1-w)\frac{\tau^k - \tau^l}{\bar{Q}^f}$$

and from Proposition 2

$$\frac{\partial\psi^i}{\partial(1/\eta)} = -(\psi^i)^2 \frac{\tau^i \lambda(1-\lambda)}{[\lambda + (1-\lambda)/\eta]^2} < 0$$

for every  $i \in (\mathbb{I} \setminus \{1\}) \cup \{m\}$ . Combining the preceding expressions gives

$$\frac{d\psi^i}{d\bar{Q}^l} = \frac{\partial\psi^i}{\partial(1/\eta)} \frac{d(1/\eta)}{d\bar{Q}^l} < 0 \quad \text{if and only if} \quad \tau^k > \tau^l.$$

□

## Proof of Proposition 5

*Proof.* We analyze the effect of increasing  $\bar{Q}^f$  for two cases: the case where there is no risky asset in the economy (i.e.,  $Q^m = 0$ ) and where there is (i.e.,  $Q^m > 0$ ). Specifically, we look at the comparative static

$$\frac{\partial\psi^i}{\partial\bar{Q}^f} = \frac{\partial\psi^i}{\partial(1/\eta)} \frac{\partial(1/\eta)}{\partial\bar{Q}^f}$$

where  $\frac{\partial\psi^i}{\partial(1/\eta)} = -(\psi^i)^2 \frac{\tau^i \lambda(1-\lambda)}{(\lambda + (1-\lambda)/\eta)^2} < 0$  for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$  by Proposition 2.

If  $Q^m = 0$ , then setting  $w = 0$  in Eq. (11), the value of convenience  $\eta$  is given by

$$\eta = \frac{1}{1 - (1 - \lambda)\bar{\tau}/2}, \quad (\text{A.7})$$

and so

$$\frac{\partial(1/\eta)}{\partial \bar{Q}^f} = \frac{(1 - \lambda)}{2} \frac{1}{\bar{Q}^f} \left( \bar{\tau} - \sum_i \frac{\bar{Q}^i}{\bar{Q}^f} \tau^i \frac{\partial \bar{Q}^i}{\partial \bar{Q}^f} \frac{\bar{Q}^f}{\bar{Q}^i} \right) = 0.$$

This shows the first part of the proposition. Next, consider the case of  $Q^m > 0$ . In this case, by Eq. (11), we have

$$\frac{\partial(1/\eta)}{\partial \bar{Q}^f} = -\frac{1 - \lambda}{2} (\tau^m - \bar{\tau}) \frac{\partial w}{\partial \bar{Q}^f} = -\frac{1 - \lambda}{2} (\tau^m - \bar{\tau}) \frac{1}{\gamma \sigma^2} \frac{\partial \pi}{\partial \bar{Q}^f}.$$

Next we find the derivative of the log risk premium with respect to  $\bar{Q}^f$ . Given the log-approximation from Section 2.4, we can find a closed form solution for the log risk premium, as we show in the following lemma:

**Lemma 5 (Log risk premium).** *Under the log-approximation of the investor's portfolio problem, the period  $t = 1$  log risk premium is given by*

$$\pi = \frac{\mu - 1 - q + \sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}}{2},$$

where  $q \equiv Q^m/\bar{Q}^f$ .

*Proof.* First, from the approximation from Section 2.4, the log risk premium on the risky asset is approximately

$$\pi \approx \log E[\varphi^m(z)] + \frac{\bar{P}_1^f - P_1^m}{P_1^m} = \mu - 1 + \frac{\bar{P}_1^f}{P_1^m}.$$

Combining this equation for the log risk premium with the equations for period  $t = 1$  equilibrium asset prices from Lemma 1 and the equation for the optimal portfolio share

$w$  from Eq. (6), we find that

$$\pi = \mu - 1 + \left( \frac{\gamma\sigma^2}{\pi} - 1 \right) \frac{Q^m}{Q^f}.$$

Therefore, we have a quadratic equation for the log risk premium of the form

$$\pi^2 - (\mu - 1 - q)\pi - \gamma\sigma^2 q = 0,$$

where  $q \equiv Q^m/\bar{Q}^f$  is the ratio of the supply of risky to risk-free assets. Notice that since the third term of the quadratic equation is negative, it follows that the equation has one positive and one negative root. Only the positive root is relevant in our case. Therefore,

$$\pi = \frac{\mu - 1 - q + \sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}}{2}.$$

□

Differentiating with respect to  $q$ , we find that

$$\frac{\partial \pi}{\partial q} = \frac{1}{2} \left( -1 + \frac{q - \mu + 1 + 2\gamma\sigma^2}{\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}} \right).$$

Consider the sign of the term in parentheses and notice that

$$(\mu - 1 - q)^2 + 4\gamma\sigma^2 q - (q - \mu + 1 + 2\gamma\sigma^2)^2 = 4\gamma\sigma^2 (\mu - 1 - \gamma\sigma^2).$$

Therefore, if  $\mu - 1 > \gamma\sigma^2$ , then

$$(\mu - 1 - q)^2 + 4\gamma\sigma^2 q > (q - \mu + 1 + 2\gamma\sigma^2)^2,$$

and so the sign of the term in parenthesis is negative, or  $\partial\pi/\partial q < 0$ . Similarly, if  $\mu - 1 \leq$

$\gamma\sigma^2$ , we can conclude that  $\partial\pi/\partial q \geq 0$ .

From the proof of Lemma 5 we know that  $\pi \approx \mu - 1 + \frac{\bar{P}_1^f}{\bar{P}_1^m}$  and from Lemma 1  $\frac{\bar{P}_1^f}{\bar{P}_1^m} = \frac{1-w}{w}q \geq 0$ . Therefore  $\mu - 1 - \gamma\sigma^2 \approx \pi - \gamma\sigma^2 - \frac{1-w}{w}q$ . The Merton share given by equation (6)  $w = \frac{\pi}{\gamma\sigma^2}$  must be less than one and therefore  $\pi - \gamma\sigma^2 \leq 0$  and  $\frac{1-w}{w} \geq 0$ . This implies that  $\mu - 1 - \gamma\sigma^2 \approx \pi - \gamma\sigma^2 - \frac{1-w}{w}q \leq 0$  and therefore  $\partial\pi/\partial q \geq 0$ .

Since  $\partial q/\partial \bar{Q}^f < 0$ , it follows that  $\partial\pi/\partial \bar{Q}^f \leq 0$ . Therefore,  $\partial(1/\eta)/\partial \bar{Q}^f \geq 0$  and  $\frac{\partial\psi^i}{\partial \bar{Q}^f} \leq 0$  iff  $(\tau^m - \bar{\tau}) \geq 0$ .  $\square$

### Proof of Proposition 6

*Proof.* Consider the derivative  $\partial\psi^i/\partial\sigma^2$  for any  $i \in \mathbb{I} \cup m$ . We have that

$$\frac{\partial\psi^i}{\partial\sigma^2} = \frac{\partial\psi^i}{\partial(1/\eta)} \frac{\partial(1/\eta)}{\partial\sigma^2},$$

where  $\frac{\partial\psi^i}{\partial(1/\eta)} = -(\psi^i)^2 \frac{\tau^i\lambda(1-\lambda)}{(\lambda+(1-\lambda)/\eta)^2} < 0$  for all  $i \in (\mathbb{I} \setminus \{1\}) \cup m$  by Proposition 2. Furthermore, by Eq. (11) we have that

$$\frac{\partial(1/\eta)}{\partial\sigma^2} = -\frac{1-\lambda}{2}(\tau^m - \bar{\tau})\frac{\partial w}{\partial\sigma^2} = \frac{1-\lambda}{2}(\tau^m - \bar{\tau})\frac{1}{\gamma\sigma^2} \left( \frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2} \right).$$

Note first that

$$\frac{\partial\pi}{\partial\sigma^2} = \frac{\gamma q}{\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q}} > 0.$$

Next, note that

$$\frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2} = \frac{(\mu - 1 - q)\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2 q} + (\mu - 1 - q)^2 + 2\gamma\sigma^2 q}{2}.$$

Suppose that  $\mu - 1 - q \geq 0$ . In that case it clearly follows that the right-hand side is positive. Suppose instead that  $\mu - 1 - q < 0$ . In that case, note that

$$\begin{aligned}
& (\mu - 1 - q)\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2q} + (\mu - 1 - q)^2 + 2\gamma\sigma^2q > \\
& -\frac{(\mu - 1 - q)^2 + \left(\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2q}\right)^2}{2} + (\mu - 1 - q)^2 + 2\gamma\sigma^2q = \\
& \frac{(\mu - 1 - q)^2 - \left(\sqrt{(\mu - 1 - q)^2 + 4\gamma\sigma^2q}\right)^2}{2} + 2\gamma\sigma^2q = \\
& -2\gamma\sigma^2q + 2\gamma\sigma^2q = 0.
\end{aligned}$$

Therefore, we have that

$$\frac{\pi}{\sigma^2} - \frac{\partial\pi}{\partial\sigma^2} > 0,$$

and so the sign of  $\frac{\partial(1/\eta)}{\partial\sigma^2}$  depends on the sign of  $\tau^m - \bar{\tau}$ .

The derivation of the comparative statics with respect to risk aversion  $\gamma$  is analogous. □

### Proof of Proposition 7

*Proof.* Define the loading  $x \equiv \frac{\lambda}{\lambda + (1-\lambda)/\eta}$ , so that by Proposition 2,  $\psi^i = \frac{1}{1-\tau^i x}$  and  $\frac{\partial\psi^i}{\partial x} = (\psi^i)^2 \tau^i > 0$  for all  $i \in \mathbb{I} \cup m$  with  $\tau^i > 0$ . Hence the sign of any comparative static of  $\psi^i$  equals the sign of the corresponding comparative static of  $x$  and is common across assets. Let  $\tau^p \equiv \bar{\tau}(1-w) + \tau^m w$  denote the portfolio-weighted average transaction cost. By Eq. (11),  $\frac{1-\lambda}{\eta} = (1-\lambda) - \frac{(1-\lambda)^2 \tau^p}{2}$ , and therefore

$$x = \frac{\lambda}{1 - \frac{(1-\lambda)^2 \tau^p}{2}}.$$

Note that under the log-approximation,  $w$  does not depend on  $\lambda$ : by Lemma 1,  $\bar{P}_1^f / P_1^m = \frac{(1-w)Q^m}{wQ^f}$ , so the log risk premium  $\pi$ , and hence the Merton share in Eq. (6), do not involve

$\lambda$ . Differentiating,

$$\frac{\partial x}{\partial \lambda} = \frac{1 - \frac{(1-\lambda)^2 \tau^p}{2} + \lambda(1-\lambda)\tau^p}{\left[1 - \frac{(1-\lambda)^2 \tau^p}{2}\right]^2} > 0,$$

since  $\tau^p \leq \max\{\bar{\tau}, \tau^m\} < 2$ . Together with  $\frac{\partial \psi^i}{\partial x} > 0$ , this shows that  $\frac{\partial \psi^i}{\partial \lambda} > 0$ .

For part i), with  $\lambda = \lambda(\sigma^2)$ , the chain rule gives

$$\frac{d\psi^i}{d\sigma^2} = \frac{\partial \psi^i}{\partial x} \left[ \frac{\partial x}{\partial \tau^p} \frac{\partial \tau^p}{\partial \sigma^2} + \frac{\partial x}{\partial \lambda} \lambda'(\sigma^2) \right].$$

Holding  $\lambda$  fixed,  $\sigma^2$  affects  $x$  only through  $\tau^p$  via the portfolio share  $w$ :

$$\frac{\partial x}{\partial \tau^p} \frac{\partial \tau^p}{\partial \sigma^2} = \frac{\partial x}{\partial \tau^p} (\tau^m - \bar{\tau}) \frac{\partial w}{\partial \sigma^2}, \quad \frac{\partial x}{\partial \tau^p} = \frac{\lambda(1-\lambda)^2/2}{\left[1 - \frac{(1-\lambda)^2 \tau^p}{2}\right]^2} > 0,$$

and, by the proof of Proposition 6,  $\frac{\partial w}{\partial \sigma^2} = -\frac{1}{\gamma \sigma^2} \left( \frac{\pi}{\sigma^2} - \frac{\partial \pi}{\partial \sigma^2} \right) < 0$ . Hence the first term,  $\frac{\partial \psi^i}{\partial \sigma^2} = \frac{\partial \psi^i}{\partial x} \frac{\partial x}{\partial \tau^p} \frac{\partial \tau^p}{\partial \sigma^2}$ , is negative iff  $\tau^m > \bar{\tau}$ , which restates Proposition 6, while the second term,  $\frac{\partial \psi^i}{\partial \lambda} \lambda'(\sigma^2)$ , is non-negative, since  $\frac{\partial \psi^i}{\partial \lambda} > 0$ . This proves part i).

For part ii), define

$$\bar{\xi} \equiv -\frac{\frac{\partial x}{\partial \tau^p} \frac{\partial \tau^p}{\partial \sigma^2}}{\frac{\partial x}{\partial \lambda}} = \frac{\frac{\lambda(1-\lambda)^2}{2} (\tau^m - \bar{\tau}) \left(-\frac{\partial w}{\partial \sigma^2}\right)}{1 - \frac{(1-\lambda)^2 \tau^p}{2} + \lambda(1-\lambda)\tau^p} > 0$$

for  $\tau^m > \bar{\tau}$ . Since  $\frac{\partial \psi^i}{\partial x} > 0$  for all  $i$ , we have  $\frac{d\psi^i}{d\sigma^2} > 0$  for every  $i \in \mathbb{I} \cup m$  if and only if  $\frac{\partial x}{\partial \tau^p} \frac{\partial \tau^p}{\partial \sigma^2} + \frac{\partial x}{\partial \lambda} \lambda'(\sigma^2) > 0$ , that is, if and only if  $\lambda'(\sigma^2) > \bar{\xi}$ . Note that  $\bar{\xi}$  does not depend on  $i$ , so the reversal applies to the entire cross-section simultaneously. The derivations with respect to the risk aversion parameter  $\gamma$  are analogous, using the comparative static of  $w$  with respect to  $\gamma$  established in the proof of Proposition 6.  $\square$

### Proof of Proposition 8

*Proof.* We can express  $c_{1,\underline{S}}$  as a function of  $\eta$ :  $c_{1,\underline{S}} = \frac{Y_1}{\frac{2\lambda}{\eta} + (1-\lambda)}$ . We insert it in the constraints and the constraints into the objective function of the social planner. This implies:

$$\begin{aligned} W \equiv & \log(Y_0) + 2\lambda \log\left(\frac{1}{\lambda}\right) \\ & + 2\lambda \log\left(Y_1 - (1-\lambda)\frac{Y_1}{\frac{2\lambda}{\eta} + (1-\lambda)}\right) + (1-\lambda) \log\left(\frac{Y_1}{\frac{2\lambda}{\eta} + (1-\lambda)}\right) \\ & + (1-\lambda) \log\left(\left(E\left[\left(\sum_i \delta^i Q^i + \varphi_2^m(z)Q^m\right)^{1-\gamma}\right]\right)^{1/(1-\gamma)}\right). \end{aligned}$$

We want to analyze the impact of  $\eta$  on  $W$ . Taking the respective first-order condition we have

$$\frac{\partial W}{\partial \eta} = \frac{(1-\lambda)}{\eta^2} (1-\eta) \frac{2\lambda}{\frac{2\lambda}{\eta} + (1-\lambda)}.$$

It follows that  $\frac{\partial W}{\partial \eta} = 0$  iff  $\eta = 1$ . We observe that for  $\eta \geq 1$ , which is the possible equilibrium outcome in our model, welfare is monotone decreasing in the value of convenience, i.e.,  $\frac{\partial W}{\partial \eta} < 0$ .  $\square$

### Proof of Propositions 9 and 10

*Proof.* We first modify Lemmas 1 and 2 and allow for buyer transaction costs/subsidies. The maximization problem of the impatient investor in period  $t = 1$  is trivial – it is optimal for her to sell all her asset holdings and consume. Therefore, for that investor we have

$$V_{1,\bar{S}} = 2 \log(c_{1,\bar{S}}),$$

where  $c_{1,\bar{S}} = Y_1 + \sum_{i \in \mathbb{I}} (1 - \tau^i) P_1^i X_1^i$ , with  $X_1^i = Q^i$  in equilibrium.

The maximization problem of the patient investor, who buys all assets in equilibrium

at  $t = 1$  in the absence of  $t = 2$  payoff risk, is now given by

$$\begin{aligned}
V_{1,\underline{S}} &= \max_{c_{1,\underline{S}}, c_{2,\underline{S}}, \{X_{2,\underline{S}}^i\}_i} \log(c_{1,\underline{S}}) + \log(c_{2,\underline{S}}) \\
&\text{s.t.} \\
c_{1,\underline{S}} + \sum_{i \in \mathbb{I}} P_1^i X_{2,\underline{S}}^i &= Y_1 + \sum_{i \in \mathbb{I}} P_1^i X_1^i - \sum_{i \in \mathbb{I}} \tau_b^i P_1^i (X_{2,\underline{S}}^i - X_1^i), \\
c_{2,\underline{S}} &= \sum_{i \in \mathbb{I}} \delta^i X_{2,\underline{S}}^i.
\end{aligned}$$

Assuming an interior solution for each asset  $i$  (which will be the case in equilibrium), the Euler equation for asset  $i$  is given by

$$\frac{1}{c_{2,\underline{S}}} = \frac{\overline{P}_1^i}{c_{1,\underline{S}}}, \quad (\text{A.8})$$

where  $\overline{P}_1^i = \frac{(1+\tau_b^i)}{\delta^i} P_1^i$ . Since these first-order conditions have to hold for each asset  $i$  in equilibrium, this implies that  $\overline{P}_1^i$  must be equalized across assets  $i$ . Therefore  $\overline{P}_1^f = \overline{P}_1^i$ . Since  $X_1^i = Q^i$  in equilibrium, market clearing implies that  $X_{2,\underline{S}}^i = \frac{1}{(1-\lambda)} Q^i$ , which we insert into the budget constraints. This yields

$$\begin{aligned}
c_{1,\underline{S}} &= Y_1 - \frac{\lambda}{(1-\lambda)} \sum_{i \in \mathbb{I}} \overline{P}_1^i \overline{Q}^i, \\
c_{2,\underline{S}} &= \frac{1}{(1-\lambda)} \sum_{i \in \mathbb{I}} \overline{Q}^i,
\end{aligned}$$

where

$$\overline{Q}^i = \delta^i Q^i.$$

Substituting for the period 1 and 2 equilibrium consumption in the Euler equation,

Eq. (A.8), and solving for the common adjusted price  $\bar{P}_1^f$ , we obtain

$$\bar{P}_1^f = \frac{1 - \lambda}{1 + \lambda} \frac{Y_1}{\sum_{i \in \mathbb{I}} \bar{Q}^i}. \quad (\text{A.9})$$

Next, we characterize the  $t = 0$  maximization problem of an investor:

$$\begin{aligned} V_0 = \max_{\{X_1^i\}_{i \in \mathbb{I}}} & \log(c_0) + \lambda V_{1,\bar{S}} + (1 - \lambda) V_{1,\underline{S}} \\ \text{s.t. } c_0 = & Y_0 + \sum_{i=1}^N P_0^i (Q^i - X_1^i). \end{aligned}$$

Taking first-order conditions, and using the Envelope Theorem for the period  $t = 1$  marginal values, we obtain the period  $t = 0$  Euler equations

$$\frac{\partial U_0}{\partial c_0} = \lambda \frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}} \frac{P_1^i}{P_0^i} (1 - \tau^i) + (1 - \lambda) \frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} \frac{P_1^i}{P_0^i} (1 + \tau_b^i).$$

Next, we derive the equilibrium value of convenience,  $\eta$ . Recall that  $\eta = \frac{m_{1,\bar{S}}}{m_{1,\underline{S}}} = \frac{\frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}}}{\frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}}}$ . We know that

$$\begin{aligned} \frac{\partial U_{1,\bar{S}}}{\partial c_{1,\bar{S}}} &= \frac{2}{c_{1,\bar{S}}}, \\ &= \frac{2}{Y_1 + \sum_{i \in \mathbb{I}} (1 - \tau^i) P_1^i X_1^i}, \\ &= \frac{2}{Y_1 + \sum_{i \in \mathbb{I}} \frac{(1 - \tau^i)}{(1 + \tau_b^i)} \bar{P}_1^f \bar{Q}^i}, \\ &= \frac{2}{Y_1 + \left[ \sum_{i \in \mathbb{I}} \frac{(1 - \tau^i)}{(1 + \tau_b^i)} \bar{Q}^i \right] \frac{(1 - \lambda)}{(1 + \lambda)} \frac{Y_1}{\sum_{i \in \mathbb{I}} \bar{Q}^i}} \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial U_{1,\underline{S}}}{\partial c_{1,\underline{S}}} &= \frac{1}{c_{1,\underline{S}}}, \\
&= \frac{1}{Y_1 - \frac{\lambda}{(1-\lambda)} \sum_{i \in \mathbb{I}} \overline{P}_1^i \overline{Q}^i}, \\
&= \frac{(1+\lambda)}{Y_1}.
\end{aligned}$$

Therefore

$$\eta = \frac{2}{1 + \lambda + (1 - \lambda) \sum_{i \in \mathbb{I}} \frac{\overline{Q}^i}{\sum_{i \in \mathbb{I}} \overline{Q}^i} \frac{(1-\tau^i)}{(1+\tau_b^i)}},$$

which we can further simplify to

$$\eta = \frac{1}{1 - \frac{(1-\lambda)}{2} \sum_{i \in \mathbb{I}} \frac{\overline{Q}^i}{\sum_{i \in \mathbb{I}} \overline{Q}^i} \frac{\tau^i + \tau_b^i}{(1+\tau_b^i)}}. \tag{A.10}$$

To derive Proposition 9 suppose that  $\tau_b^i = 0, \forall i$ . In that case, Eq. (A.10) simplifies to

$$\eta = \frac{1}{1 - \frac{(1-\lambda)}{2} \sum_{i \in \mathbb{I}} \frac{\overline{Q}^i}{\sum_{i \in \mathbb{I}} \overline{Q}^i} \tau^i}.$$

Direct inspection of this equation shows that by increasing  $\delta^1$ , the payoff on the most liquid asset, and at the same time decreasing the payoffs  $\delta^i$  of the other assets, to ensure budget balance, one can decrease the value of  $\eta$ , i.e.  $\left. \frac{\partial \eta}{\partial \delta^1} \right|_{\sum_i \delta^i Q^i = Q^f} < 0$ .

To derive Proposition 10 we inspect Eq. (A.10). If  $\tau_b^i = -\tau^i, \forall i$ , then  $\eta = 1$ .  $\square$

## **B Data Appendix**

### **B.1 Data sources and summary statistics**

We obtain government security yields and OIS rates from Bloomberg (see Table B.1). We use it to calculate the Treasury-OIS spreads. The maturities we use are 3 months, 1 year, 2 years, 5 years, 7 years, and 10 years. For all maturities except the 10 year swap, we use the Fed Funds OIS rate. For the 10 year maturity, we use the SOFR OIS rate because the corresponding Fed Funds OIS series contains numerous missing observations prior to 2013. Over the overlapping sample period, the average difference between the SOFR OIS and Fed Funds OIS rates is only 5 basis points. The MOVE Index also comes from Bloomberg (ticker: MOVE Index), and the VIX Index is obtained from FRED. Lastly, we obtain the Effective Federal Funds Rate from FRED. The frequency of all data is daily. For the regressions, we convert the data to weekly frequency by retaining the last business-day observation of each week. Table B.1 gives an overview of the Bloomberg tickers of the used data series for the Treasury yields and OIS rates.

Table B.1: Data sources

| Data     | Maturity | Source    | Ticker          |
|----------|----------|-----------|-----------------|
| Treasury | 3m       | Bloomberg | USGG3M Index    |
|          | 1y       | Bloomberg | USGG1Y Index    |
|          | 2y       | Bloomberg | USGG2Y Index    |
|          | 5y       | Bloomberg | USGG5Y Index    |
|          | 7y       | Bloomberg | USGG7Y Index    |
|          | 10y      | Bloomberg | USGG10Y Index   |
| OIS      | 3m       | Bloomberg | USSOC Curncy    |
|          | 1y       | Bloomberg | USSO1 Curncy    |
|          | 2y       | Bloomberg | USSO2 Curncy    |
|          | 5y       | Bloomberg | USSO5 Curncy    |
|          | 7y       | Bloomberg | USSO7 Curncy    |
|          | 10y      | Bloomberg | USOSFR10 Curncy |

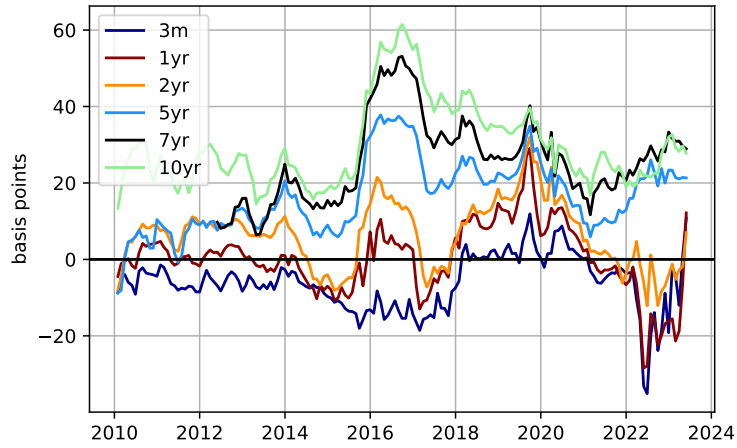
Table B.2: Summary statistics US

| Data                             | Maturity | Mean  | Median | St. Dev. |
|----------------------------------|----------|-------|--------|----------|
| Treasury-OIS<br>spread in bp     | 1y       | 6.65  | 6.15   | 6.75     |
|                                  | 2y       | 12.83 | 12.19  | 8.34     |
|                                  | 5y       | 22.95 | 19.10  | 13.10    |
|                                  | 7y       | 32.55 | 24.54  | 15.28    |
|                                  | 10y      | 36.26 | 31.81  | 14.32    |
| MOVE Index                       |          | 75.05 | 70.00  | 23.41    |
| VIX Index                        |          | 18.51 | 16.77  | 7.10     |
| Effective Fed Funds<br>Rate in % |          | 0.76  | 0.16   | 1.09     |

*Notes:* The time horizon used for the summary statistics is 2010 to mid-May 2023. We convert the data to weekly frequency by retaining the last business-day observation of each week.

## B.2 Additional figures

Figure B.1: Treasury-OIS spread



*Notes:* This figure shows the US Treasury-OIS spread across different maturities. Source: Bloomberg and authors' calculations.